Power System Robust Dispatch

A dynamic robust optimization framework

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- Static robust optimization (Soyster, 1973)

$$\begin{array}{c} \min_{x \in X} \; c^T x \\ s.t.\; Ax \le b \\ \forall b \in W \end{array} \Rightarrow \begin{array}{c} \min_{x \in X} \; c^T x \\ s.t.\; A_i x \le \min_{b \in W} b_i, \forall i \end{array}$$

- x should be made before b is known exactly without violating system constraints for arbitrary $b \in W$
- Static RO is over pessimistic
- Static RO cannot deal with "$=$" constraints

$$\begin{array}{c} x = b \\ \forall b \in [0,1] \end{array} \Rightarrow \begin{array}{c} x \le \min_{b \in [0,1]} b = 0 \\ x \ge \max_{b \in [0,1]} b = 1 \end{array}$$

Mathematical model

- Adjustable robust optimization (Ben-Tal, 2004)

$$\min_{x \in X} \{c^T x \mid \forall b \in W, \ \exists y : By \le b - Ax\}$$

- x is the first stage decisions, which must be made before uncertain data b is known, in order to recover system constraints, y can be adjusted with respect to the actual value of b
- ARO is more flexible, but generally NP-hard
- If impose affine feedback policy $y = Gb$, ARO can be reduced to a linear program.

Equivalent linear program

- Assume the uncertainty set W is a polytope

$$W = \{b \,|Hb \leq h\}$$

- Robust feasibility constraints

$$Ax + By \leq b, \forall b \in W, y = Gb$$
$$\Downarrow$$
$$A_i x + \max_{b \in W} (BG - I)_i b \leq 0, \forall i$$
$$\Downarrow$$
$$A_i x + \gamma_i^T h \leq 0, \gamma_i \geq 0, \gamma_i^T H = (BG - I)_i, \forall i$$

- The equivalent linear program

$$\min_{x,\Gamma,G}\{c^T x | x \in X, Ax + \Gamma h \leq 0, \Gamma \geq 0, \Gamma H = BG - I\}$$

Mathematical model

- Adaptive robust optimization (Bertsimas, 2011)

$$\min_{x \in X} c^T x + \max_{b \in W} \min_{y \in Y(x,b)} d^T y$$
$$Y(x,b) = \{y \mid By \le b - Ax\}$$

- Considers the worst-case second stage cost
- Affine assumption is relaxed
- The second-stage problem is NP-hard
- Solved by a Benders decomposition algorithm

Solution Methodology

- The vertices of W is vert$(W) = \{b^l\}, l = 1, 2, \cdots$
- An equivalent formulation

$$\begin{aligned} &\min c^T x + \tau \\ s.t.\ x \in X,\ \tau \geq d^T y^l,\ &Ax + By^l \leq b^l, \forall b^l \in \text{vert}(W) \end{aligned} \tag{I}$$

- A decomposition algorithm (C&CG, Zeng, 2013)
  Step 1: Set $LB = -\infty, UB = +\infty$, vert$(W) = \emptyset$.
  Step 2: Solve master problem (I), $LB = c^T x^* + \tau^*$.
  Step 3: Solve subproblem $\max\limits_{b \in W} \min\limits_{y \in Y(x^*, b)} d^T y$,
  $$UB = \min\{UB, c^T x^* + d^T y^*\}$$
  Step 4: If $UB - LB \leq \varepsilon$, terminate, deploy x^*, otherwise, vert$(W) =$ vert$(W) \cup b^*$, go to Step 2.

Solution Methodology

- BLP reformulation for the subproblem (Falk, 1973)

$$\max_{b \in W} \min_{y \in Y(x,b)} d^T y = \max_{b \in W, u \in U} u^T (b - Ax)$$

$$Y(x,b) = \{y | By \le b - Ax\}$$

$$U = \{u | B^T u = d, u \le 0\}$$

- Consider a cardinality constrained uncertainty set

$$W = \left\{ b \;\middle|\; \begin{array}{c} b_i = b_i^e + b_i^h (z_i^+ - z_i^-), \forall i \\ \{z^+, z^-\} \in Z \end{array} \right\}$$

$$Z = \left\{ \{z^+, z^-\} \;\middle|\; \begin{array}{c} z^+, z^- \in \{0,1\}^{N_W} \\ z^+ + z^- \le 1 \\ 1^T (z^+ + z^-) \le \Delta \end{array} \right\}$$

Solution Methodology

- Linearizing the objective

$$u_i b_i = u_i b_i^e + b_i^h (u_i z_i^+ - u_i z_i^-) = u_i b_i^e + b_i^h (v_i^+ - v_i^-)$$

- MILP reformulation for the subproblem with cardinality constrained uncertainty set

$$\begin{gathered}
\max_{u,v^+,v^-,z^+,z^-} u^T b^e + (v^+ - v^-)^T b^h \\
s.t. \quad u \in U, z \in Z \\
0 \le v_i^+ - u_i \le M(1 - z_i^+), \forall i \\
-M z_i^+ \le v_i^+ \le 0, \forall i \\
0 \le v_i^- - u_i \le M(1 - z_i^-), \forall i \\
-M z_i^- \le v_i^- \le 0, \forall i
\end{gathered}$$

Mathematical model

- Pre-dispatch (first-stage decision variable)
  - p_i^f Scheduled output in the nominal scenario
  - r_i^+ Upward spinning reserve capacity
  - r_i^- Downward spinning reserve capacity
- Real-time dispatch (Second-stage decision variable)
  - p_i^+ Upward regulation power
  - p_i^- Downward regulation power
- Decide the optimal generation schedule p^f, spinning reserve $\{r^+, r^-\}$, such that for arbitrary actual wind realization $w \in P^W$, the real-time dispatch is feasible, the worst-case total cost is minimized.

Mathematical model

- Uncertainty of wind generation

$$P^W = \left\{ w \;\middle|\; \begin{array}{c} w_j = w_j^e + w_j^h (z_j^+ - z_j^-), \forall j \\ \{z^+, z^-\} \in Z^W \end{array} \right\}$$

$$Z^W = \left\{ \{z^+, z^-\} \;\middle|\; \begin{array}{c} z_j^+, z_j^- \in \{0, 1\}, \forall j \\ z_j^+ + z_j^- \leq 1, \forall j \\ \sum_j (z_j^+ + z_j^-) \leq \Delta \end{array} \right\}$$

- In the worst-case wind generation scenario, the wind power can reach the upper or lower bounds in at most Δ wind farms

Mathematical model

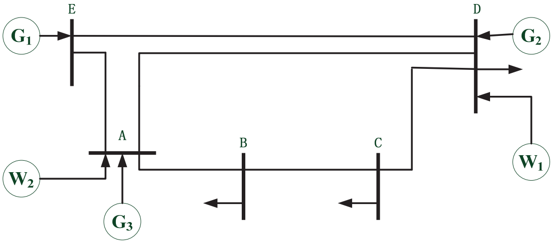
$$\min_{\{p^f, r^+, r^-\} \in X} \sum_i \left(b_i p_i^f + d_i^+ r_i^+ + d_i^- r_i^- \right) + \max_{\Delta w \in P^W} \min_{\{p^+, p^-\} \in Y} \sum_i \left(b_i^+ p_i^+ + b_i^- p_i^- \right)$$

$$X = \left\{ \{p^f, r^+, r^-\} \,\middle|\, \begin{array}{c} P_i^u + r_i^- \le p_i^f \le P_i^u - r_i^+ \ \forall i \\ \sum_i p_i^f + \sum_j w_j^e = \sum_q p_q \\ -F_l \le \sum_i \pi_{il} p_i^f + \sum_j \pi_{jl} w_j^e \\ - \sum_q \pi_{ql} p_q \le F_l, \forall l \\ 0 \le r_i^- \le R_i^- \Delta t, 0 \le r_i^+ \le R_i^+ \Delta t, \forall i \end{array} \right\}$$

$$Y = \left\{ \{p^+, p^-\} \,\middle|\, \begin{array}{c} \sum_i (p_i^f + p_i^+ - p_i^-) + \sum_j (w_j^e + \Delta w_j) = \sum_q p_q \\ -F_l \le \sum_i \pi_{il} (p_i^f + p_i^+ - p_i^-) + \\ \sum_j \pi_{jl} (w_j^e + \Delta w_j) - \sum_q \pi_{ql} p_q \le F_l, \forall l \\ 0 \le p_i^+ \le r_i^+, 0 \le p_i^- \le r_i^-, \forall i \end{array} \right\}$$

PJM 5-bus system

- System topology



- Parameter of generators

Unit No.	P_{min}/P_{max} MW	d^+/d^- CNY/MWh	b^+/b^- CNY/MWh	R^+/R^- MW/h
G1	[150, 400]	200	300	40
G2	[200, 500]	300	450	50
G3	[250, 600]	360	520	60

- Parameter of transmission lines

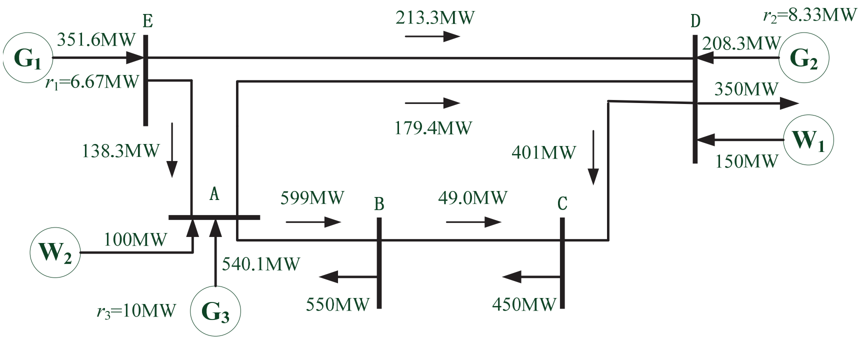
Line	From Node	To Node	Reactance	Capacity (MW)
L1	A	B	0.0281	600
L2	A	D	0.0304	300
L3	A	E	0.0064	200
L4	B	C	0.0108	100
L5	C	D	0.0297	401
L6	D	E	0.0297	300

- Uncertainty of wind generation

$$P^W = \left\{ w \,\middle|\, \begin{matrix} 135 \le w_1 \le 165 \\ 90 \le w_2 \le 110 \end{matrix} \right\}$$

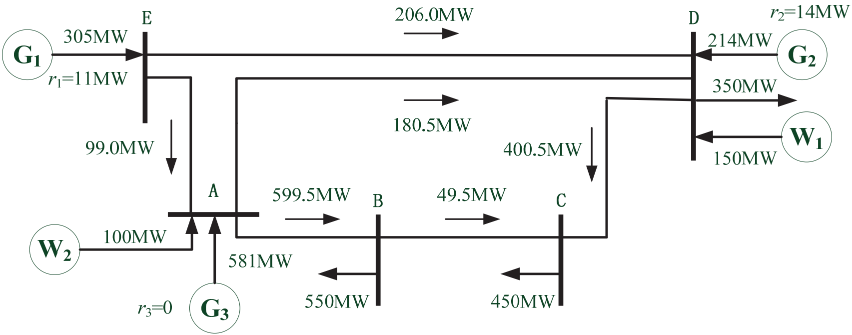
Results of traditional ERD

- Reserve is assigned proportionally
- Power flow in Line CD reaches its limit



Results of robust ERD

- Reserve is assigned to G_1 and G_2
- Certain margin is preserved in all lines



Definition

- Dispatchability W^D is the largest region in the uncertainty space, such that all the elements in it will not cause infeasibility in real-time dispatch (second stage problem) with fixed generation schedule p^f and spinning reserve $\{r^+, r^-\}$ (first-stage decision).

$$Y(x,w) = \{y \mid By \le b - Ax - Cw\}$$

$$W^D = \{w \mid Y(x,w) \neq \emptyset\}$$

$$x = \{p^f, r^+, r^-\}, y = \{p^+, p^-\}$$

- A projection form

$$W^D = \text{Proj}_w(P), P = \{w, y \mid By + Cw \le b - Ax\}$$

- Theorem: W^D is the following polytope

$$W^D = \{w \mid u^T C w \geq u^T (b - Ax), \forall u \in \text{vert}(U)\}$$

$$U = \{u | B^T u = 0, -1 \leq u \leq 0\}$$

- A delayed constraint generation algorithm
  Step 1: Choose a sufficiently large set W^D
  Step 2: Solve the following problem

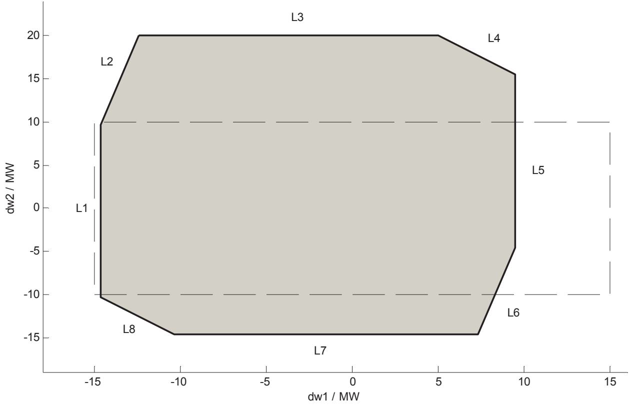
$$r = \max_{u \in U, w \in W^D} u^T (b - Ax - Cw)$$

  Step 3: If $r^* = 0$ terminate, report current W^D; else add the following constraint to W^D, and go to step 2

$$(u^*)^T C w \geq (u^*)^T (b - Ax)$$

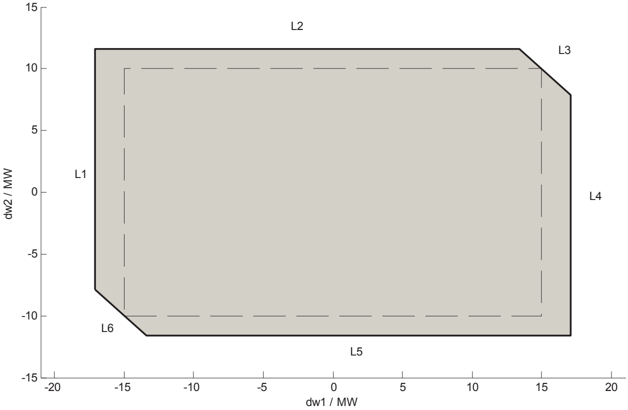
PJM 5-bus system

- W^D of traditional ERD does not cover the uncertainty set, the RTD is infeasible when $w \in P^W - W^D$



PJM 5-bus system

- W^D of robust ERD covers the entire uncertainty set, moreover, when $w \notin P^W$, RTD may still be feasible



Thanks!