Method for improving the accuracy of state estimation as new opportunity for multi-energy networks applications

Dr Dragan Cetenovic, Research Associate, The University of Manchester, Manchester, UK

Associate Professor, University of Kragujevac, Serbia

Actual Situation and Problems

Main differences:

	Transmission	Distribution
Redundancy	High	Low
Dominant measurements	Real time	Pseudo
Three phase system	Balanced	Unbalanced
No. of nodes	Low-Medium	High
Ratio R/X	Low	High

Solution:

- Forecasting Aided State Estimator

Kalman Filter Parameters:

- Initialy estimated state vector $\boldsymbol{x}_0^+$
- Covariance matrix of initaial state estimates $\boldsymbol{P}_0^+$
- Process noise covariance matrix $\boldsymbol{Q}_k$

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Process Model

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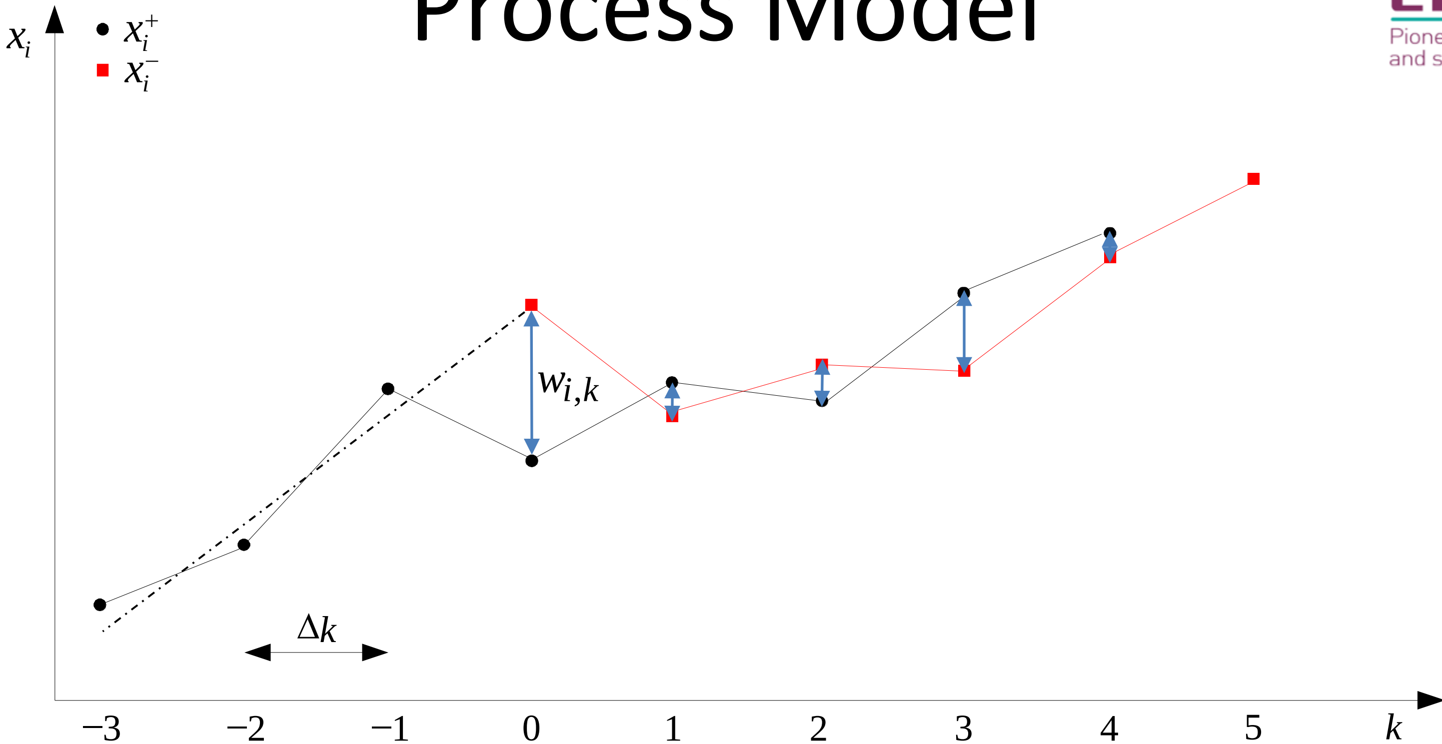
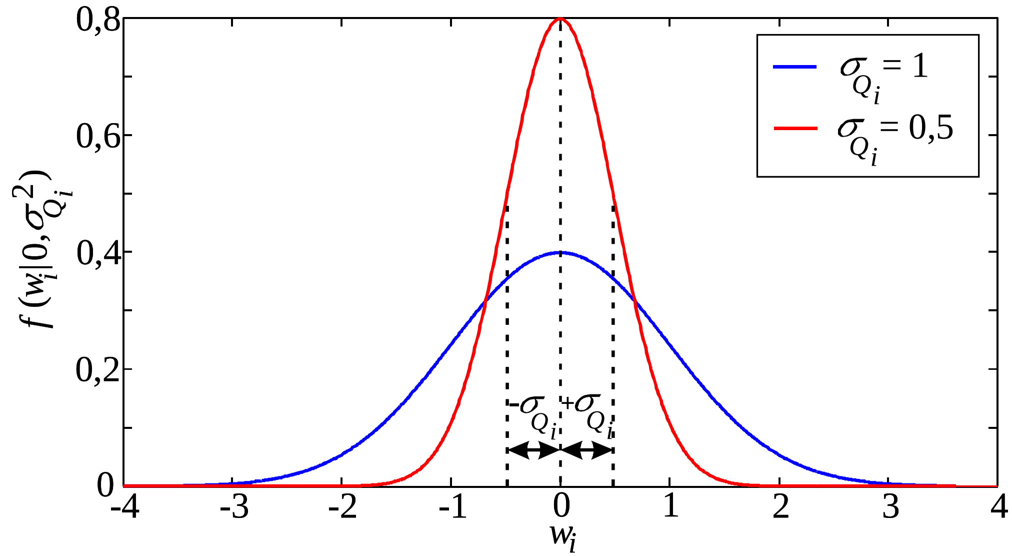


State transition of a single state variable

$$x^{-}_{i,k+1} = F_{i,k} x^{+}_{i,k} + g_{i,k}$$

$$x^{+}_{i,k+1} = x^{-}_{i,k+1} + w_{i,k}$$

$$w_{i,k} \sim \mathcal{N}\left(0, \sigma^2_{Q\,i,k}\right)$$



Gaussian distribution

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Process Model

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System state transition

$$\boldsymbol{x}_{k+1}^{-} = \boldsymbol{F}_k \boldsymbol{x}_k^{+} + \boldsymbol{g}_k$$

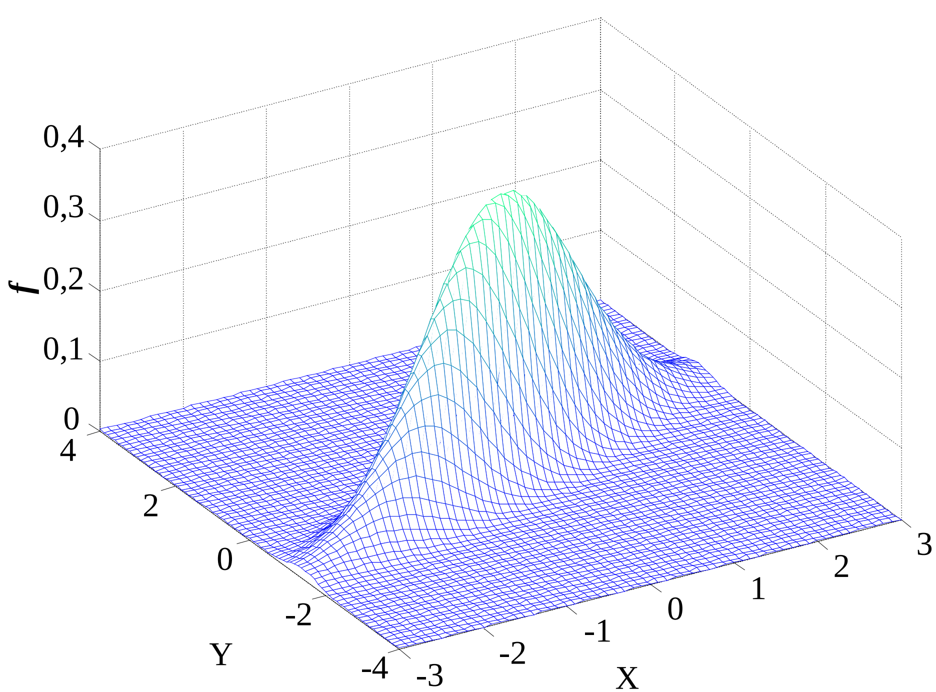
$$\boldsymbol{x}_{k+1}^{+} = \boldsymbol{x}_{k+1}^{-} + \boldsymbol{w}_k$$

$$\boldsymbol{w}_k \sim \mathcal{N}(\boldsymbol{0}, \boldsymbol{Q}_k)$$

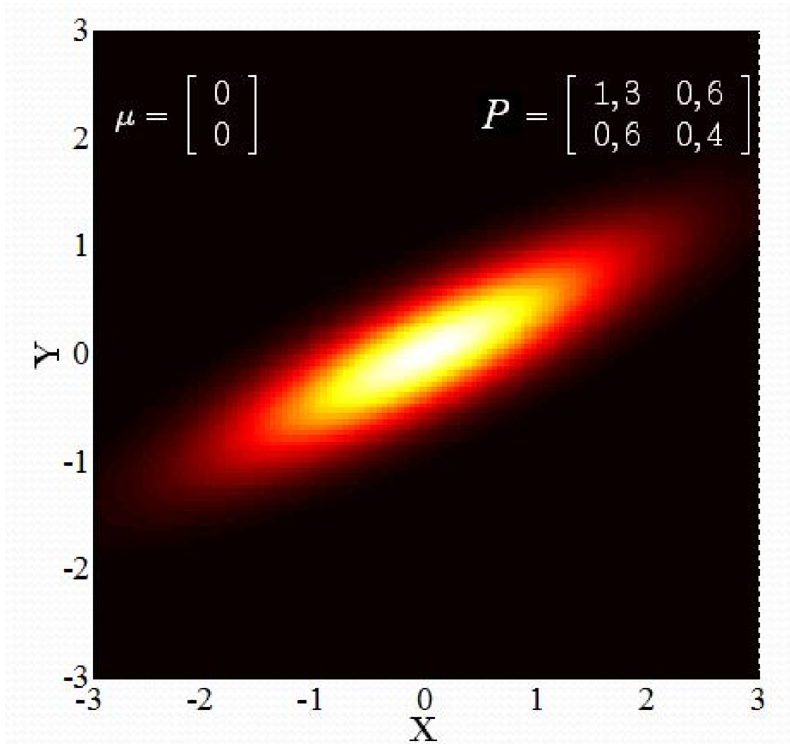
Process noise covariance matrix

$$\boldsymbol{Q}_k = \begin{bmatrix} \sigma_{Q_{1,k}}^2 & \cdots & \rho_{1i}\cdot\sigma_{Q_{1,k}}\cdot\sigma_{Q_{i,k}} & \cdots & \rho_{1n}\cdot\sigma_{Q_{1,k}}\cdot\sigma_{Q_{n,k}} \\ \vdots & \ddots & \vdots & \vdots & \vdots \\ \rho_{1i}\cdot\sigma_{Q_{1,k}}\cdot\sigma_{Q_{i,k}} & \cdots & \sigma_{Q_{i,k}}^2 & \cdots & \rho_{in}\cdot\sigma_{Q_{1,k}}\cdot\sigma_{Q_{n,k}} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho_{1n}\cdot\sigma_{Q_{1,k}}\cdot\sigma_{Q_{n,k}} & \cdots & \rho_{in}\cdot\sigma_{Q_{1,k}}\cdot\sigma_{Q_{n,k}} & \cdots & \sigma_{Q_{n,k}}^2 \end{bmatrix}$$

3-D



2-D



Measurements in Electric Distribution Networks

Real time measuremetns

-Convetional measuremetns

- Bus voltage magnitudes
- Active and reactive branch power flows
- Active and reactive bus power injections
- Branch current flows
- Bus current injections

-PMU measurements

- Bus volatage phasors
- Bus current phasors
- Branch current phasors

Pseudo measurements

- Active and reactive power injections

Virtual measurements

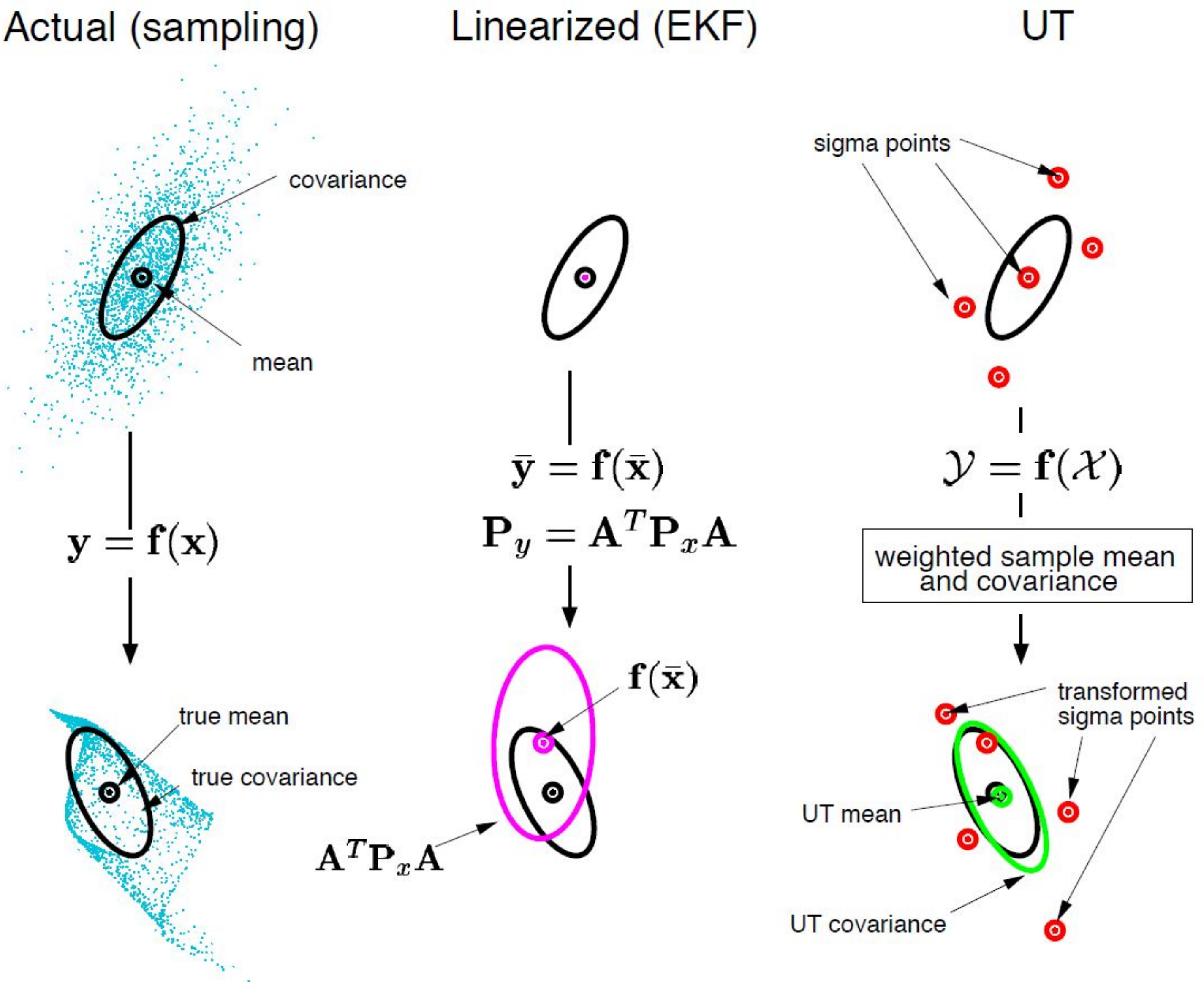
- Zero bus injections

Measurement model

$$\boldsymbol{z}_k = \boldsymbol{h}(\boldsymbol{x}_k) + \boldsymbol{e}_k$$

$$\boldsymbol{e}_k \sim \mathcal{N}(\boldsymbol{0},\ \boldsymbol{R}_k)$$

EKF vs UKF



Propagation of two-dimensional radnom variable through non-linear function

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First order EKF

1. Parameter identification

$$\alpha, \beta, \mathbf{Q}, \mathbf{R}, \mathbf{x}_0^+, \mathbf{P}_0^+, \mathbf{a}_{-1}, \mathbf{b}_{-1}$$

2. Forcesting

$$\mathbf{x}_{k+1}^- = \mathbf{F}_k \mathbf{x}_k^+ + \mathbf{g}_k$$

$$\mathbf{P}_{k+1}^- = \mathbf{F}_k \mathbf{P}_k^+ \mathbf{F}_k^{\mathrm{T}} + \mathbf{Q}_k$$

3. Filtering

$$\mathbf{z}_{k+1}^- = \mathbf{h}\left(\mathbf{x}_{k+1}^-\right)$$

$$\mathbf{T}_{k+1} = \mathbf{H}_{k+1} \mathbf{P}_{k+1}^- \mathbf{H}_{k+1}^{\mathrm{T}}$$

$$\mathbf{v}_{k+1} = \mathbf{z}_{k+1} - \mathbf{z}_{k+1}^-$$

$$\mathbf{S}_{k+1} = \mathbf{T}_{k+1} + \mathbf{R}_{k+1}$$

$$\mathbf{K}_{k+1} = \mathbf{P}_{k+1}^- \mathbf{H}_{k+1}^{\mathrm{T}} \mathbf{S}_{k+1}^{-1}$$

$$\mathbf{x}_{k+1}^+ = \mathbf{x}_{k+1}^- + \mathbf{K}_{k+1} \mathbf{v}_{k+1}$$

$$\mathbf{P}_{k+1}^+ = \mathbf{P}_{k+1}^- - \mathbf{K}_{k+1} \mathbf{S}_{k+1} \mathbf{K}_{k+1}^{\mathrm{T}}$$

UKF

1. Parameter identification

$$\alpha, \beta, \mathbf{Q}, \mathbf{R}, \mathbf{x}_0^+, \mathbf{P}_0^+, \mathbf{a}_{-1}, \mathbf{b}_{-1}, \Big| \alpha_{ut}, \beta_{ut}, \kappa_{ut}$$

2. Forecasting

$$\mathbf{Y}_k^+ = \mathbf{x}_k^+ \cdot \mathbf{1}^{\mathrm{T}} + \sqrt{n+\lambda_{ut}} \begin{bmatrix} \mathbf{0} & \sqrt{\mathbf{P}_k^+} & -\sqrt{\mathbf{P}_k^+} \end{bmatrix}$$

$$\hat{\mathbf{X}}_{k+1} = \mathbf{F}_k \mathbf{Y}_k^+ + \mathbf{g}_k \cdot \mathbf{1}^{\mathrm{T}}$$

$$\mathbf{x}_{k+1}^- = \hat{\mathbf{X}}_{k+1} \mathbf{w}_m$$

$$\mathbf{P}_{k+1}^- = \hat{\mathbf{X}}_{k+1} \mathbf{W} \hat{\mathbf{X}}_{k+1}^{\mathrm{T}} + \mathbf{Q}_k$$

3. Filtering

$$\mathbf{Y}_{k+1}^- = \mathbf{x}_{k+1}^- \cdot \mathbf{1}^{\mathrm{T}} + \sqrt{n+\lambda_{ut}} \begin{bmatrix} \mathbf{0} & \sqrt{\mathbf{P}_{k+1}^-} & -\sqrt{\mathbf{P}_{k+1}^-} \end{bmatrix}$$

$$\hat{\mathbf{Z}}_{k+1}^- = \mathbf{h}\left(\mathbf{Y}_{k+1}^-\right)$$

$$\mathbf{z}_{k+1}^- = \hat{\mathbf{Z}}_{k+1}^- \mathbf{w}_m$$

$$\mathbf{T}_{k+1} = \hat{\mathbf{Z}}_{k+1}^- \mathbf{W} \left[\hat{\mathbf{Z}}_{k+1}^-\right]^{\mathrm{T}}$$

$$\mathbf{v}_{k+1} = \mathbf{z}_{k+1} - \mathbf{z}_{k+1}^-$$

$$\mathbf{S}_{k+1} = \mathbf{T}_{k+1} + \mathbf{R}_{k+1}$$

$$\mathbf{C}_{k+1} = \mathbf{Y}_{k+1}^- \mathbf{W} \left[\hat{\mathbf{Z}}_{k+1}^-\right]^{\mathrm{T}}$$

$$\mathbf{K}_{k+1} = \mathbf{C}_{k+1} \mathbf{S}_{k+1}^{-1}$$

New Method for Assessment of Process Noise Covariance Matrix

- **One parameter based diagonal covariance matrix *Q***

$$\boldsymbol{Q}(q) = 10^{q} \cdot \boldsymbol{I}_n$$

- **Two basic steps:**

1. estimation for assumed time-invariant parameter *q* value
2. parameter identification based on cost function minimization

- **Performance index**

- Root mean square error

$$\xi_{\tilde{n}}(q) = \frac{1}{K} \sum_{k=1}^{K} \xi_{\tilde{n},k}(q)$$

$$\xi_{\tilde{n},k}(q) = \sqrt{\frac{1}{\tilde{n}} \sum_{i=1}^{\tilde{n}} \left(x_{i,k}^{+}(q) - x_{i,k}^{true} \right)^2}$$

Cost Function

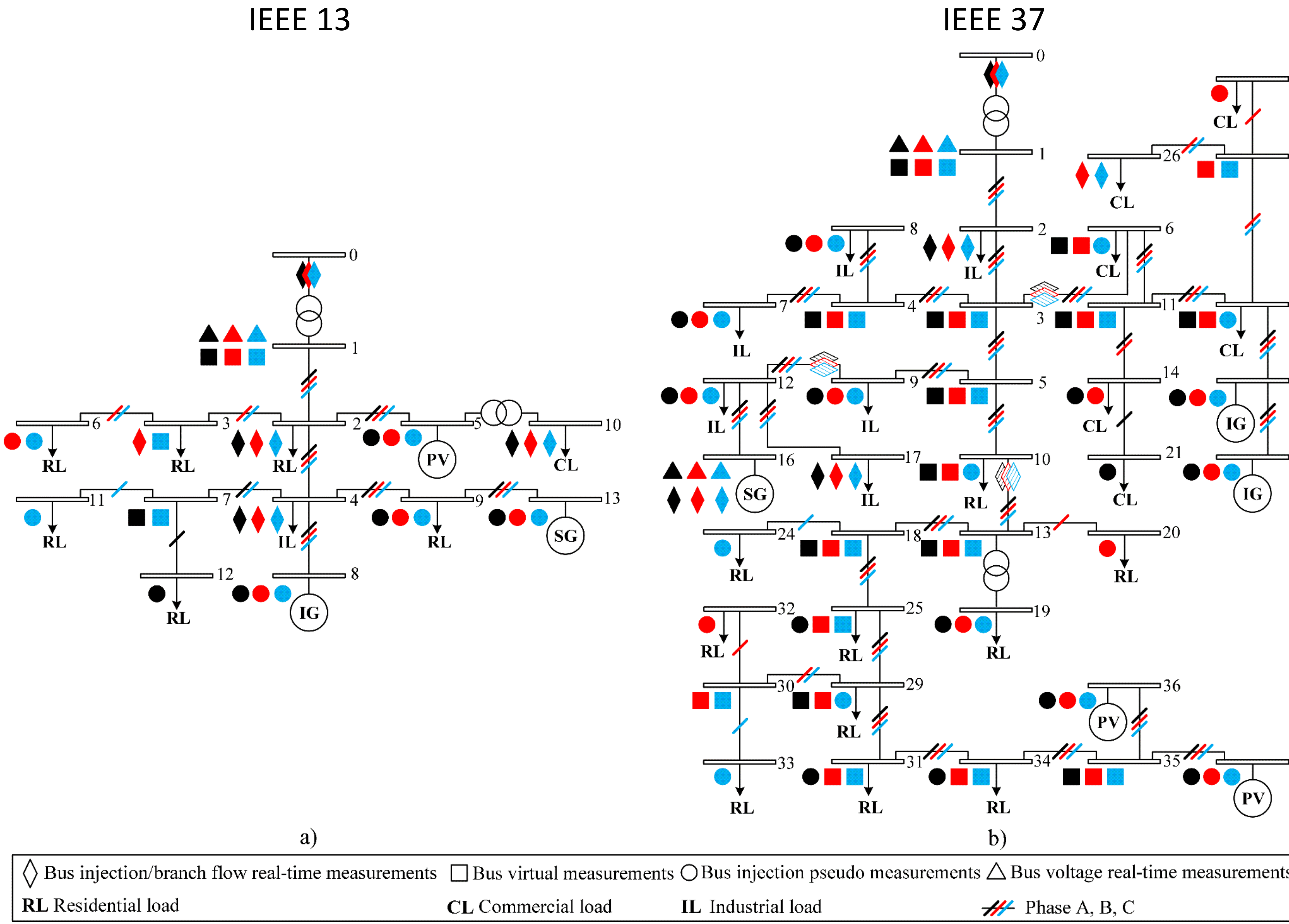
New cost function based on ARMS (*Average Root Mean Square*)

Assumptions:

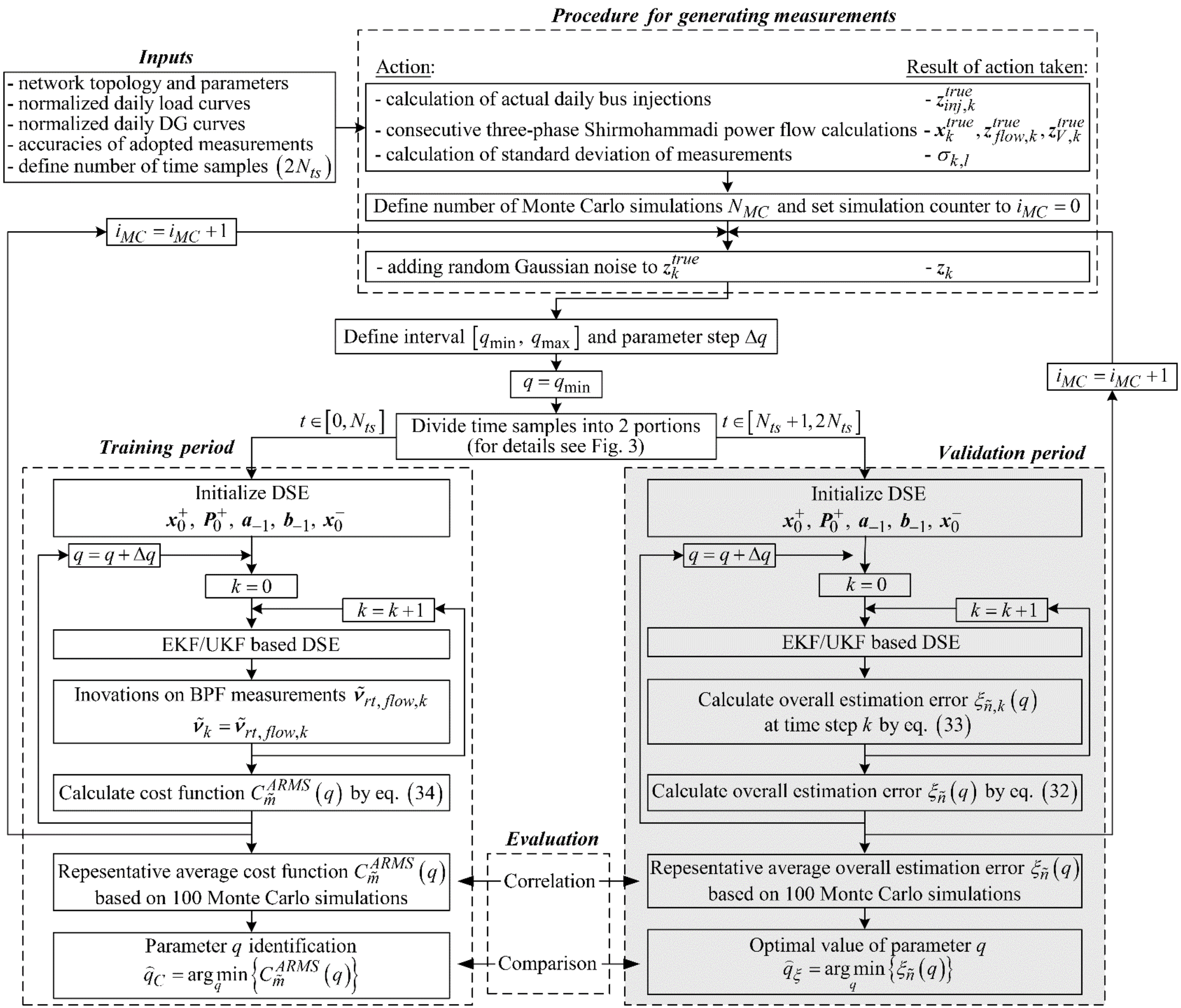
- if we define the cost function in the same form as the state estimation error, a good correlation can be established between them,
- innovations of some types of measurements are in better correlation with state estimation error then others

$$C_{\tilde{m}}(q) = C_{\tilde{m}}^{ARMS}(q) = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{1}{\tilde{m}}\sum_{l=1}^{\tilde{m}} v_{l,k}^{2}(q)} = \frac{1}{K}\sum_{k=1}^{K}\sqrt{\frac{\tilde{\boldsymbol{v}}_k^{\mathrm{T}}(q)\,\tilde{\boldsymbol{v}}_k(q)}{\tilde{m}}}$$

Test Systems



Flow Chart



Procedure for generating measurements
Inputs
- network topology and parameters
- normalized daily load curves
- normalized daily DG curves
- accuracies of adopted measurements
- define number of time samples $(2N_{ts})$
Action:
Result of action taken:
- calculation of actual daily bus injections
- $z_{inj,k}^{true}$
- consecutive three-phase Shirmohammadi power flow calculations
- $x_k^{true}, z_{flow,k}^{true}, z_{V,k}^{true}$
- calculation of standard deviation of measurements
- $\sigma_{k,l}$
Define number of Monte Carlo simulations N_{MC} and set simulation counter to $i_{MC} = 0$
$i_{MC} = i_{MC} + 1$
- adding random Gaussian noise to z_k^{true}
- z_k
Define interval $[q_{\min}, q_{\max}]$ and parameter step Δq
$q = q_{\min}$
$t \in [0, N_{ts}]$
Divide time samples into 2 portions (for details see Fig. 3)
$t \in [N_{ts}+1, 2N_{ts}]$
Training period
Validation period
Initialize DSE
$x_0^+, P_0^+, a_{-1}, b_{-1}, x_0^-$
$q = q + \Delta q$
$k = 0$
$k = k + 1$
EKF/UKF based DSE
Inovations on BPF measurements $\tilde{v}_{rt,flow,k}$
$\tilde{v}_k = \tilde{v}_{rt,flow,k}$
Calculate cost function $C_{\tilde{m}}^{ARMS}(q)$ by eq. (34)
Calculate overall estimation error $\xi_{\tilde{n},k}(q)$ at time step k by eq. (33)
Calculate overall estimation error $\xi_{\tilde{n}}(q)$ by eq. (32)
Evaluation
Repesentative average cost function $C_{\tilde{m}}^{ARMS}(q)$ based on 100 Monte Carlo simulations
Correlation
Repesentative average overall estimation error $\xi_{\tilde{n}}(q)$ based on 100 Monte Carlo simulations
Parameter q identification
$\hat{q}_C = \arg\min_q \{C_{\tilde{m}}^{ARMS}(q)\}$
Comparison
Optimal value of parameter q
$\hat{q}_\xi = \arg\min_q \{\xi_{\tilde{n}}(q)\}$

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Sensitivity analysis

Initialization:

1. „*SSE* start“

$$x_0^+ = x_0^{SSE}$$

$$P_0^+ = P_0^{SSE}$$

$$\left(P_0^+ = \left[H_0^T R_0^{-1} H_0 \right]^{-1} \right)$$

2. „true start 1“

$$x_0^+ = x_0^{true}$$

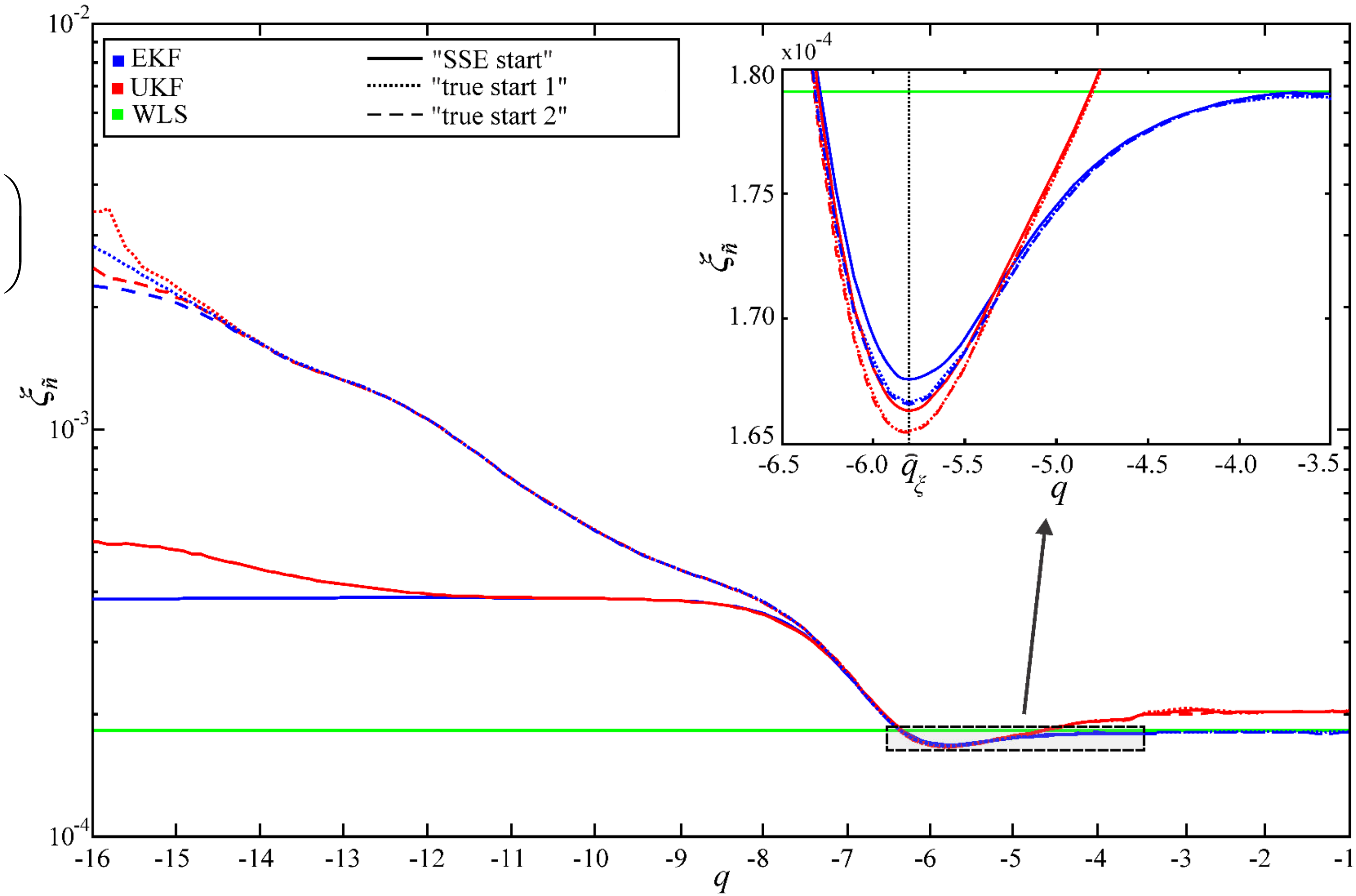
$$P_0^+ = 0_{n \times n}$$

3. „true start 2“

$$x_0^+ = x_0^{true}$$

$$P_0^+ = p_0 \cdot I_n$$

$$p_0 = 10^{-15}$$



Overall estimation error as function of parameter *q* on training period for different initialization solutions and filtering methods

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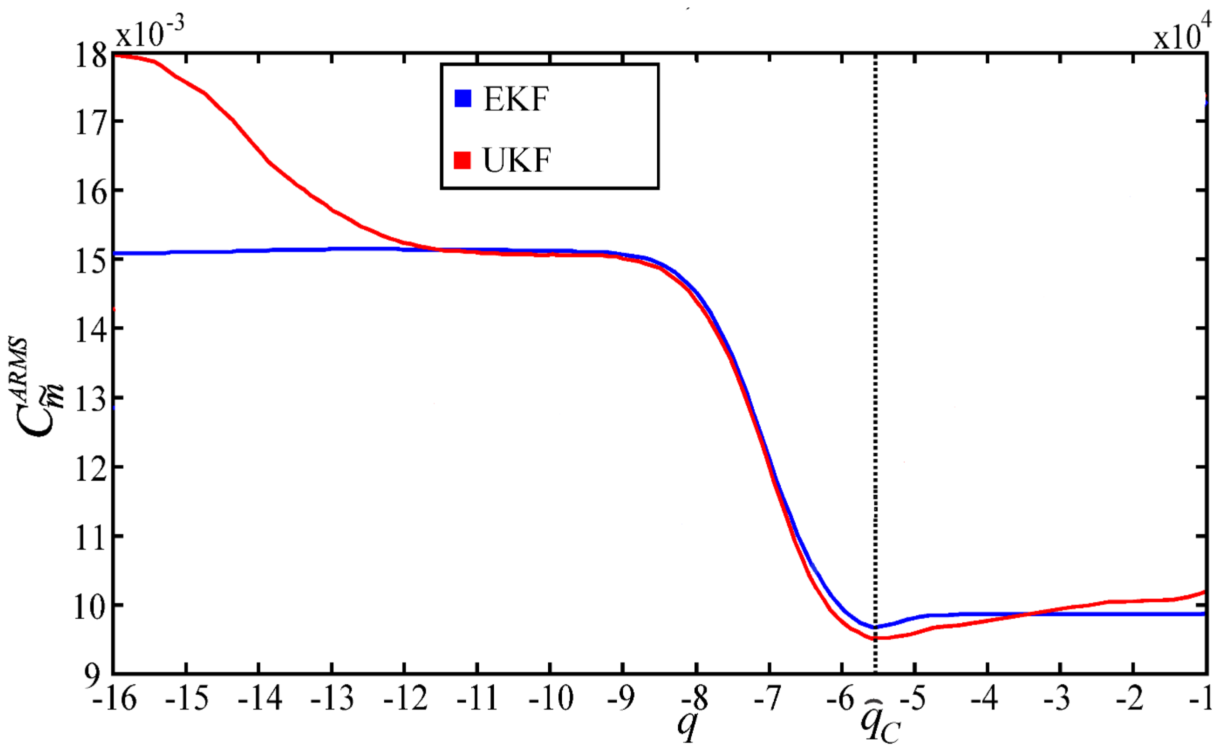
Identification of process noise covariance parameter



$\hat{q}_C \approx \hat{q}_\xi$

Cost function of BPF measurement innovations relative to parameter *q*
(Training period)

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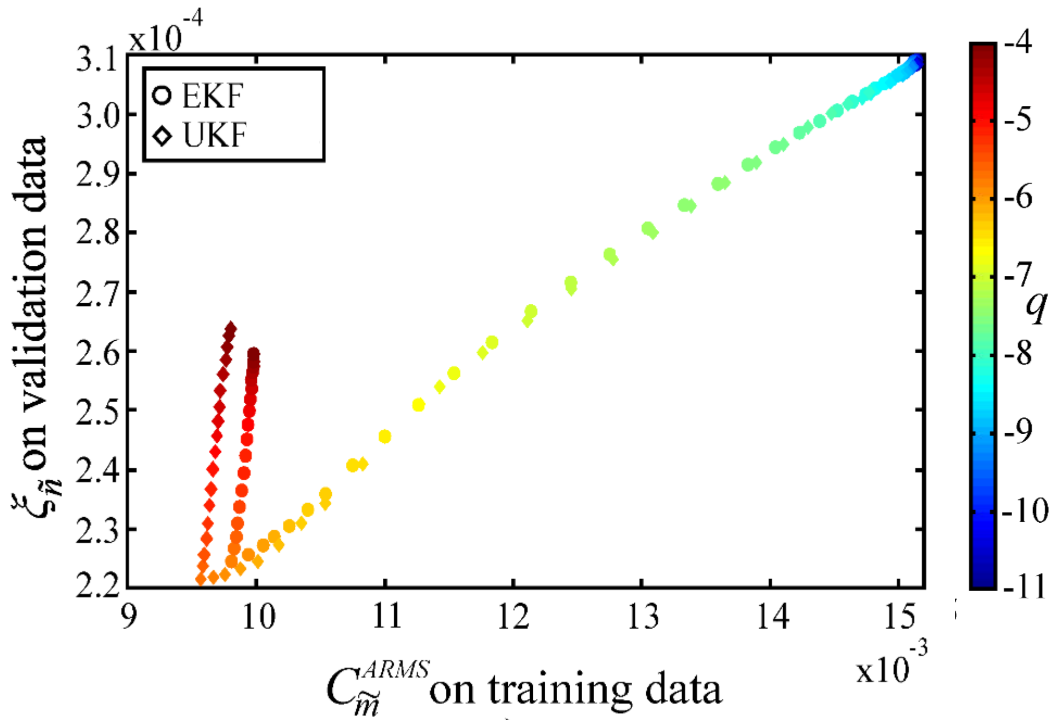
Verification of identified value



Overall stimation error (Training period) vs. Cost function applied on BPF measurement innovations (Validation period). Colorbar indicate parameter *q* value

Contributions

- More accurate state estimation
- Applicable to EKF and UKF
- Practical feasibility:

  - requests only available data (observed and predicted BPF measurements)

  - can be applied to all DNs

  - handles non-linear model

Further queries

- Sudden changes
- ME systems (electricity, gas, heating) are non-linear
- SE of ME systems is static

Thank you!