Dynamic Economic Dispatch: Feasible and Optimal Solutions

Shandong University

School Of EE

Han Xueshan

xshan@sdu.edu.cn

1 Introduction

In the operation of electric power systems, deficiency in megawatt (MW) generation to meet the load demand can be developed if the MW generation is not dispatched properly. The problem occurs because of the limitation of the unit’s ramp rate and its regulation capacity. These dynamic constraints are intertemporal coupled. Hence, the operational decision at one time interval can affect the operational decision at later time intervals. The problem is difficult to solve because of its large dimension in both time space and the number of units involved.

Dynamic economic dispatch (DED) tries to minimize the total generation cost of a given set of units while considering the corresponding constraints. The DED model can be formulated as below:

$$C_T = \sum_{t=1}^{T} \sum_{i \in N} C_i(P_i^t)$$

$$s.t. \quad \sum_{i \in N} P_i^t = D^t \qquad t = 1, 2, ..., T$$

$$P_i^{\min} \le P_i^t \le P_i^{\max} \qquad i \in N; t = 1, 2, ..., T$$

$$-DR_i \Delta t \le P_i^{t+1} - P_i^t \le UR_i \Delta t \qquad i \in N; t = 1, 2, ..., T-1$$

The spinning reserve constraints can also be considered. For brevity, they are not shown here.

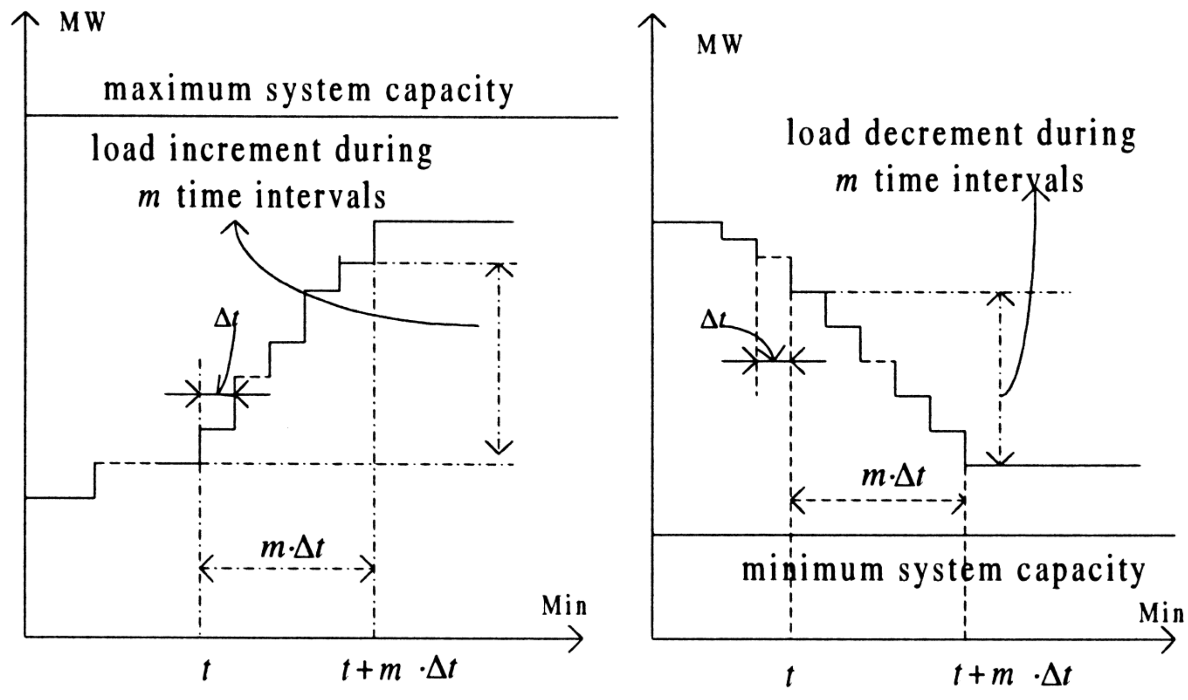
Dynamic economic dispatch (DED) has been recognized as an important function that should be properly modeled for system production cost calculations. The key to the DED problem is how to prepare the generation well ahead of time to meet the future load demand. Since 1980 this problem has become a focus of attention.

Wood used an efficient price-based algorithm to compute the generation based on the unit's minimum/maximum generation limits that are recursively determined from the computed generation at the adjacent time intervals and ramp rate limits. On the basis, Lee et al. proposed a price-based ramp rate model where ramp rate constraints are dualized and the marginal ramp rate values are determined by an iterative algorithm. Ross and Kim et al. solved the dispatch problem using dynamic programming, linear programming, non linear programming.

Although the DED's solution methods as mentioned above are efficient, they are more complicated than ED. ED, in general, can also consider the ramp rate limits for the current time interval. But it does not have the look-ahead capability for the future time intervals. If the load increase for future time intervals is too steep such that it exceeds the ramp rate capability of the less expensive units, then the more expensive units should increase their MW generation in advance. The MW increase should be adjusted such that the ramp rate limits are not violated while satisfying the system load demand. If this optimal condition can be found, the DED problem can be solved like an ED.

Our Study try to illustrates how the unit's MW generation is restrained by the ramp rate constraints during system operation. It explains how this constrained generation can be handled effectively and efficiently in solving the DED problem. The employed technique is a look-ahead algorithm that restricts the MW assignment in ED to satisfy load increase in the future time intervals. The proposed algorithm can be used in the dispatch for both the vertically integrated utility and competitive electricity market.

2. Maximum System Ramp MW



(a) (b)

The Figure above shows the load demand curve as a function of *m* time intervals. Each time interval is Δt long. The load demand is assumed to be constant during each time interval. Because of the generators' ramp rate limits and their maximum and minimum MW capacity, a maximum system MW generation exists that can be increased/decreased during the *m* time intervals. This is called maximum system ramp-up/down MW in our study.

◆ Maximum system ramp-up/down regulation capacity from period *t* to period *t+m*:

$$MSU^{m} = \sum_{i \in N_{up}^{fast}} (P_i^{\max} - P_i^{\min}) + \sum_{i \in N_{up}^{slow}} mUR_i \Delta t$$

$$MSD^{m} = \sum_{i \in N_{down}^{fast}} (P_i^{\max} - P_i^{\min}) + \sum_{i \in N_{down}^{slow}} mDR_i \Delta t$$

For any m, if

$$D^{t+m} - D^{t} > MSU^{m} \quad or \quad D^{t} - D^{t+m} > MSD^{m}$$

the DED will be infeasible.

Otherwise, there is some room to optimize the generation dispatch.

- Number of intervals for unit *i* to ramp up or down its entire operating range:

$$NU_i = ceil(\frac{P_i^{\max} - P_i^{\min}}{UR_i \Delta t})$$

$$ND_i = ceil(\frac{P_i^{\max} - P_i^{\min}}{DR_i \Delta t})$$

◆ Four units can be defined based on the given NU_i and ND_i when looking forward from period t to period $t+m$:

$$N_{up}^{fast} = \{i \mid NU_i \le m\} \qquad N_{up}^{slow} = \{i \mid NU_i > m\}$$

$$N_{down}^{fast} = \{i \mid ND_i \le m\} \qquad N_{down}^{slow} = \{i \mid ND_i > m\}$$

The definition of the fast/slow units is related to the value of m. As the increase of m, the number of fast units will increase.

The fast units can operate from their minimum output to their maximum output within m intervals to contribute their entire regulation capacity.

The slow units can increase or decrease their output at full speed.

- Unit unavailable MW of unit *i* for increasing/decreasing output from period *t* to period *t*+*m*:

$$UMU_i^{t,m} = \max(0, P_i^{\max} - P_i^t - mUR_i\Delta t)$$

$$UMD_i^{t,m} = \max(0, P_i^t - P_i^{\min} - mDR_i\Delta t)$$

3 A feasible technique for solving DED

The feasible technique solves DED problem by computing a sequence of static economic dispatch (SED) with look-ahead constraints.

The ramp rate constraints in DED are well respected in SED by adding the look-ahead constraints into the model.

Formalizing the proposed model, some definitions are introduced above.

The proposed feasible technique for solving DED is shown below:

$$loop(\ t = 1 : T$$

$$C_t = \sum_{i \in N} C_i(P_i^t)$$

$$s.t. \quad \begin{cases} \sum_{i \in N} P_i^t = D^t \\ P_i^{\min} \le P_i^t \le P_i^{\max} & i \in N; \\ -DR_i \Delta t \le P_i^{t+1} - P_i^t \le UR_i \Delta t & i \in N; \\ \text{Look-ahead constraints} \end{cases}$$

$$)$$

The look-ahead constraints include:

$$\begin{cases} \sum_{i \in N_{up}^{fast}} (P_i^t - P_i^{\min}) \le MSU^m - (D^{t+m} - D^t) & m = 1,2,\ldots,\max_i NU_i; \\ \sum_{i \in N_{down}^{fast}} (P_i^{\max} - P_i^t) \le MSD^m - (D^t - D^{t+m}) & m = 1,2,\ldots,\max_i ND_i; \\ \sum_{i \in N_{up}^{slow}} UMU_i^{t,m} \le \sum_i P_i^{\max} - D^{t+m} & m = 1,2,\ldots,\max_i NU_i; \\ \sum_{i \in N_{up}^{slow}} UMD_i^{t,m} \le D^{t+m} - \sum_i P_i^{\min} & m = 1,2,\ldots,\max_i NU_i; \end{cases}$$

- The first constraint above indicates that the fast units tend to operate near their lower limits if the system ramp-up capacity is insufficient or the demand increment is too large;
- The second constraint indicates that the fast units tend to operate near their upper limits if the system ramp-down capacity is insufficient or the demand decrement is large;
- The third and fourth constraints indicate that the slow units tend to address the demand variation first on the premise that they can adjust their output at full speed.

The proposed feasible technique decouples the DED problem and solved it interval by interval.

For each interval the optimization belongs to a SED problem in which the look-ahead constraints are considered.

The experience shown that the accuracy of the proposed feasible technique is desirable during most of the cases.

4 A optimal technique for solving DED

Define

$$NO = \max_i (NU_i, ND_i)$$

It can be found that all unit can cover their entire operating range from interval t to interval *t+NO*.

Ramping constraints do not couple intervals separated by more than NO.

The proposed optimal technique for solving DED is shown below:

$$loop(\ t = 1 : t_{final}$$

$$C_T = \sum_{t=t}^{t+NO} \sum_{i \in N} C_i(P_i^t)$$

$$s.t. \quad \sum_{i \in N} P_i^t = D^t \qquad t = t, t+1, \ldots, t+NO$$

$$P_i^{\min} \le P_i^t \le P_i^{\max} \qquad i \in N;\ t = t, t+1, \ldots, t+NO$$

$$-DR_i \Delta t \le P_i^{t+1} - P_i^t \le UR_i \Delta t \qquad i \in N;\ t = t, t+1, \ldots, t+NO$$

$$)$$

And $t_{final} = T - NO$

The proposed optimal technique decouples the DED problem and solved it by computing a sequence of sub-DED problems in which the optimization interval considered is smaller than the entire optimization interval.

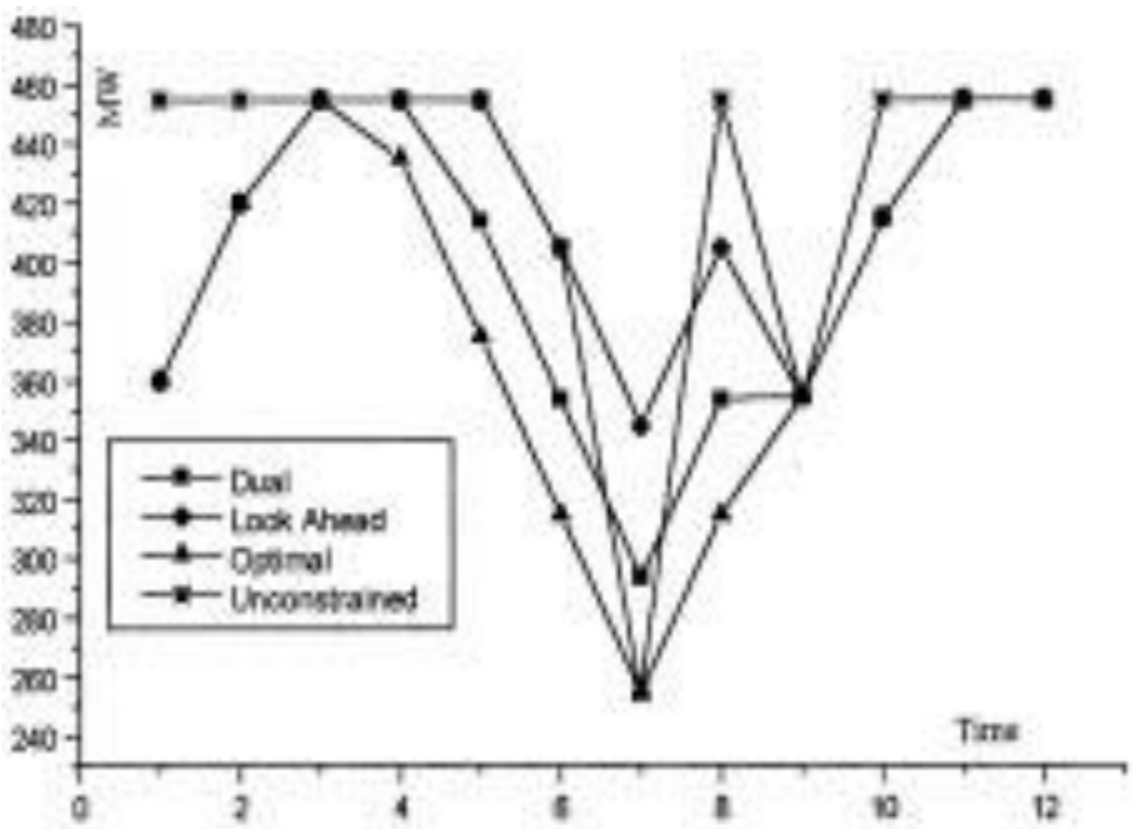
The optimal technique can obtain the globally optimal solution of the DED problem.

5 Example

F. N. Lee, L. Lemonidis, and K. C. Liu, "Price-based ramp-rate model for dynamic dispatch and unit commitment," *IEEE Trans. on Power ystems*, vol. 9, no. 3, 1994. (5 Units System)

Hour	1	2	3	4	5	6	7	8
MW	360	420	455	455	455	405	345	405
Hour	9	10	11	12	13	14	15	16
MW	355	415	455	455	455	455	455	455
Hour	17	18	19	20	21	22	23	24
MW	455	405	345	405	355	415	455	455

Comparison of the output of unit 1 of the 5-unit system for the various solution methods.



COMPARISON OF THE COSTS OF THE VARIOUS SOLUTIONS FOR THE 5-UNIT SYSTEM.

Solution	Cost	Relative cost
Unconstrained	168683.4	0.998557
Look-ahead	168982.6	1.000329
Dual	168970.4	1.000256
Optimal	168927.1	1.000000

5 CONCLUSION

Dynamic Economic Dispatch is a complex optimization problem whose importance may increase as competition in power generation intensifies. This study has attempted to clarify the techniques that provide feasible solutions. It has also presented two new solution techniques. The first is guaranteed to find a feasible solution for all load profiles. The second is an efficient technique for finding the optimal solution. Tests results have been used to demonstrate the effectiveness of these techniques and to compare their results with those obtained using previously published methods.

Thank you！