PSS design - some recent results

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A probability-one homotopy method for parameter tuning

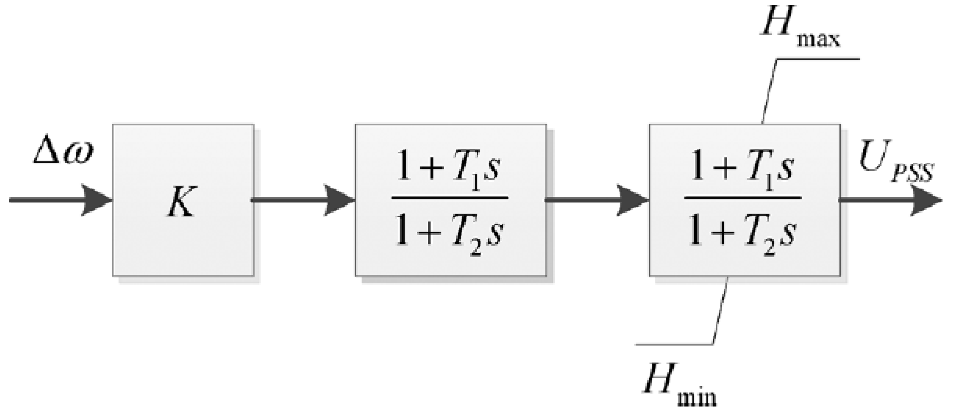
Standard pole assignment formulation

- A power system is modeled as:

$$\dot{\mathbf{X}} = \mathbf{AX} + \mathbf{BU}$$

$$\mathbf{Y} = \mathbf{CU}$$

- The standard two-phase PSS model is applied



$$\dot{\mathbf{X}}_{PSS} = \mathbf{A}_{PSS}\mathbf{X}_{PSS} + \mathbf{B}_{PSS}\mathbf{Y}$$
$$\mathbf{U}_{PSS} = \mathbf{C}_{PSS}\mathbf{X}_{PSS} + \mathbf{D}_{PSS}\mathbf{Y}$$

$$\bar{\mathbf{A}} = \begin{bmatrix} \mathbf{A} + \mathbf{BD}_{PSS}\mathbf{C} & \mathbf{BC}_{PSS} \\ \mathbf{B}_{PSS}\mathbf{C} & \mathbf{A}_{PSS} \end{bmatrix}$$

The probability-one homotopy theory

- A **pole assignment problem** can be viewed as a root-finding problem as follows:

$$\boldsymbol{\phi}(\mathbf{z}) = \lambda_{\mathbf{x}}(\bar{\mathbf{A}}(\mathbf{z})) - \lambda_{\mathbf{x0}} = \mathbf{0}, \boldsymbol{\phi}: \mathbf{R}^p \to \mathbf{R}^p$$

- Let the homotopy map be given by

$$\boldsymbol{\rho}(\mu, \mathbf{z}) = \mu\boldsymbol{\phi}(\mathbf{z}) + (1-\mu)(\mathbf{z} - \mathbf{z}_0)$$

$\boldsymbol{\rho}(0, \mathbf{z}) = \mathbf{z} - \mathbf{z}_0$ provides an initial solution $\mathbf{z}_0$.

$\boldsymbol{\rho}(1, \mathbf{z}) = \boldsymbol{\phi}(\mathbf{z})$ is what we are looking for.

- **A nice theoretical property**: for almost all $\mathbf{z}_0$, there exists a zero curve of $\boldsymbol{\rho}(\mu, \mathbf{z})$ and the Jacobian matrix of $\boldsymbol{\rho}(\mu, \mathbf{z})$ has full rank.

The probability-one homotopy theory

- Let the zero curve be parameterized by arc length t ($0 \le t \le \bar{t}$, $\bar{t}$ is the point satisfying $\mu(t) = 1$)

$$\boldsymbol{\rho}(\mu(t), \mathbf{z}(t)) = 0$$

- The following initial value problem of implicit ordinary differential equations (ODE) can be obtained:

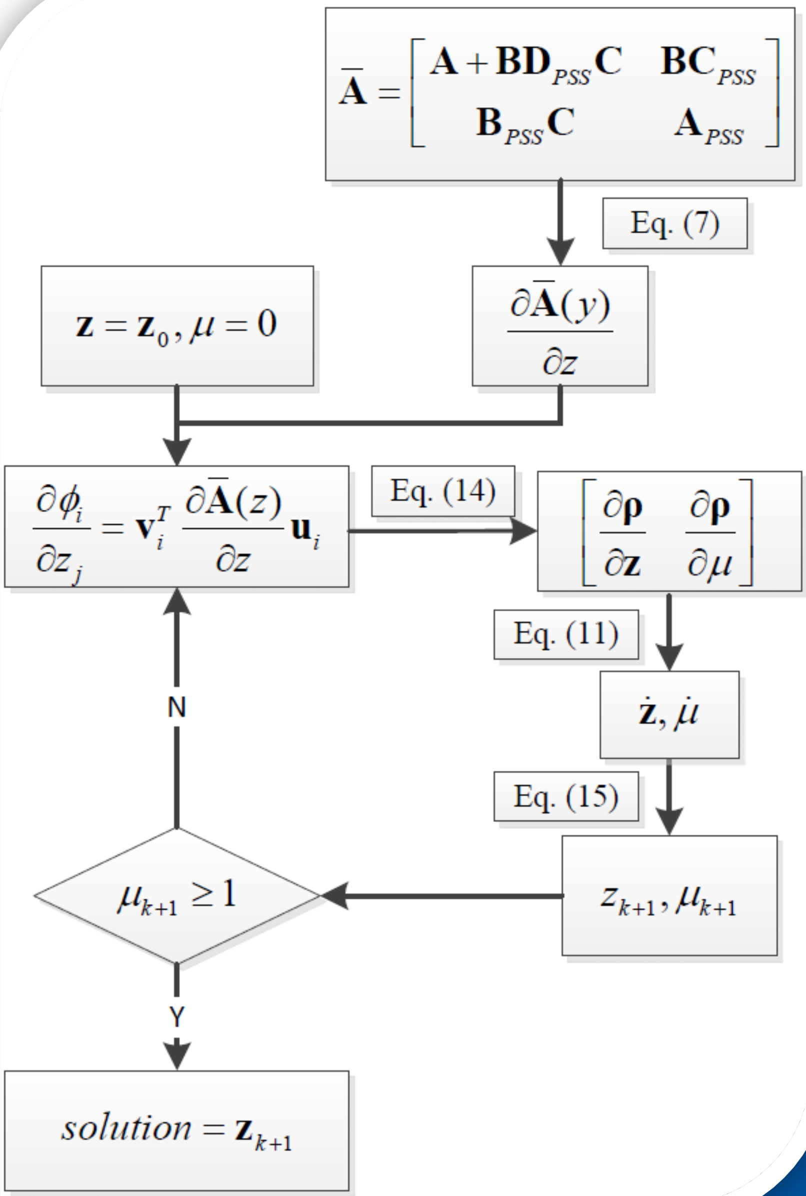
$$\begin{bmatrix} \frac{\partial \boldsymbol{\rho}}{\partial \mathbf{z}} & \frac{\partial \boldsymbol{\rho}}{\partial \mu} \end{bmatrix} \begin{bmatrix} \dot{\mathbf{z}} \\ \dot{\mu} \end{bmatrix} = 0$$

$$\left\| \dot{\mu}, \dot{\mathbf{z}} \right\|_2 = 1$$

$$\mu(0) = 0, \mathbf{z}(0) = \mathbf{z}_0$$

- To obtain this curve, the numerical integration method can be used to get the exact point $\mu(\bar{t}) = 1$.

The probability-one homotopy theory



- The previous PHM is a root-finding method, to improve the method, a **least square** formulation has also been tested.

Implementation techniques

Tracking eigenvalues	• Permute the eigenvalues obtained during iterations such that $\sum_i \lvert \lambda_i^* - \lambda_{P(i)} \rvert$ is minimized.
Computing step size	• $\Delta t = \Delta\, \mu / \dot{\mu}$
Utilizing intermediate solutions	• Quite often a single run cannot find the final solution. • If the current run does not converge, a second run can take the best intermediate solution as the initial guess.
Choosing a suitable homotopy	• Take a more general homotopy function: $\boldsymbol{\rho}(\mu, \mathbf{z}) = \mu\phi(\mathbf{z}) + (1-\mu)\mathbf{G}(\mathbf{z})$.
Rescaling	• Bad scaling of $\bar{\mathbf{A}}$ can make $\mathbf{K}$ un-optimized. • To remedy, the gain vector $\mathbf{K}$ is scaled down to the same order of magnitude as $\mathbf{T}_1$.

Simulation

- 10 test systems (from SMIB to 3296-bus) are used.
- The relationship between the **chosen system** and the most representative **implementation techniques** are summarized as:

System	PHM Techniques
Single-machine-infinity bus System (SMIB)	Basic procedure of the PHM
4-bus, 9-bus, 57-bus and 118-bus systems	Eigenvalues tracking
11-bus system	Rescaling
39-bus system	2^{nd} derivative
157-bus system	Utilizing intermediate solutions

Simulation – Advantages of PHM

- It is shown that Newton’s method required a starting point which is very close to the final solution, while **the PHM is more reliable and robust to the initial points**.

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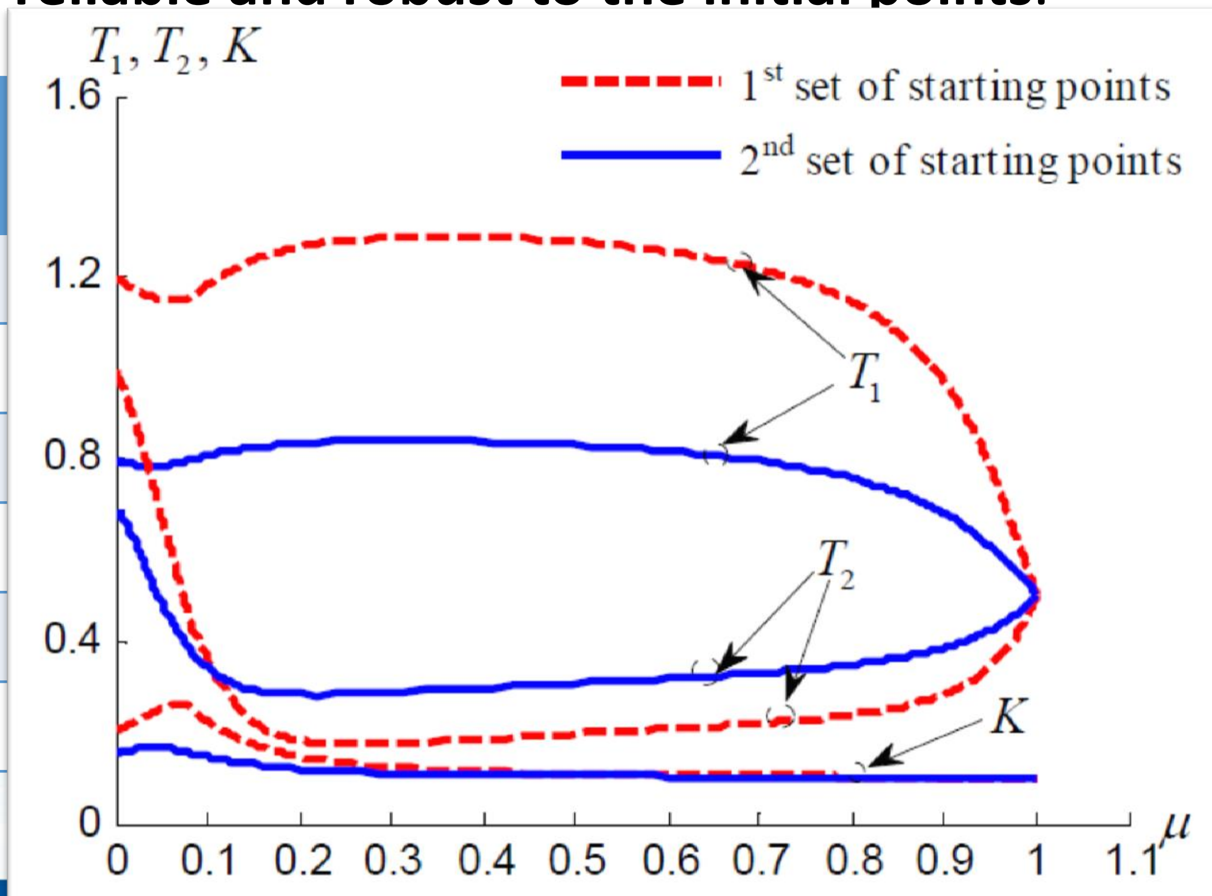
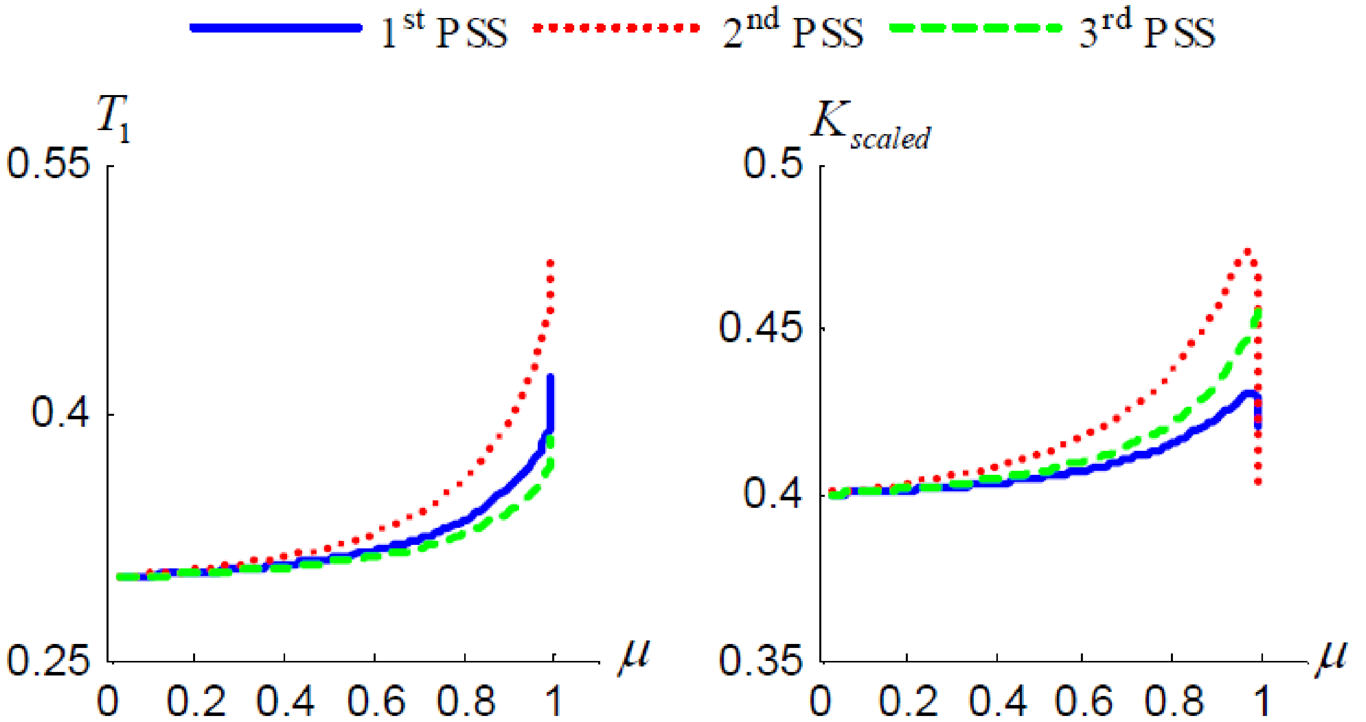
3	[illegible]
4	[illegible]
5	[illegible]
6	[illegible]



Applicability to real-world systems

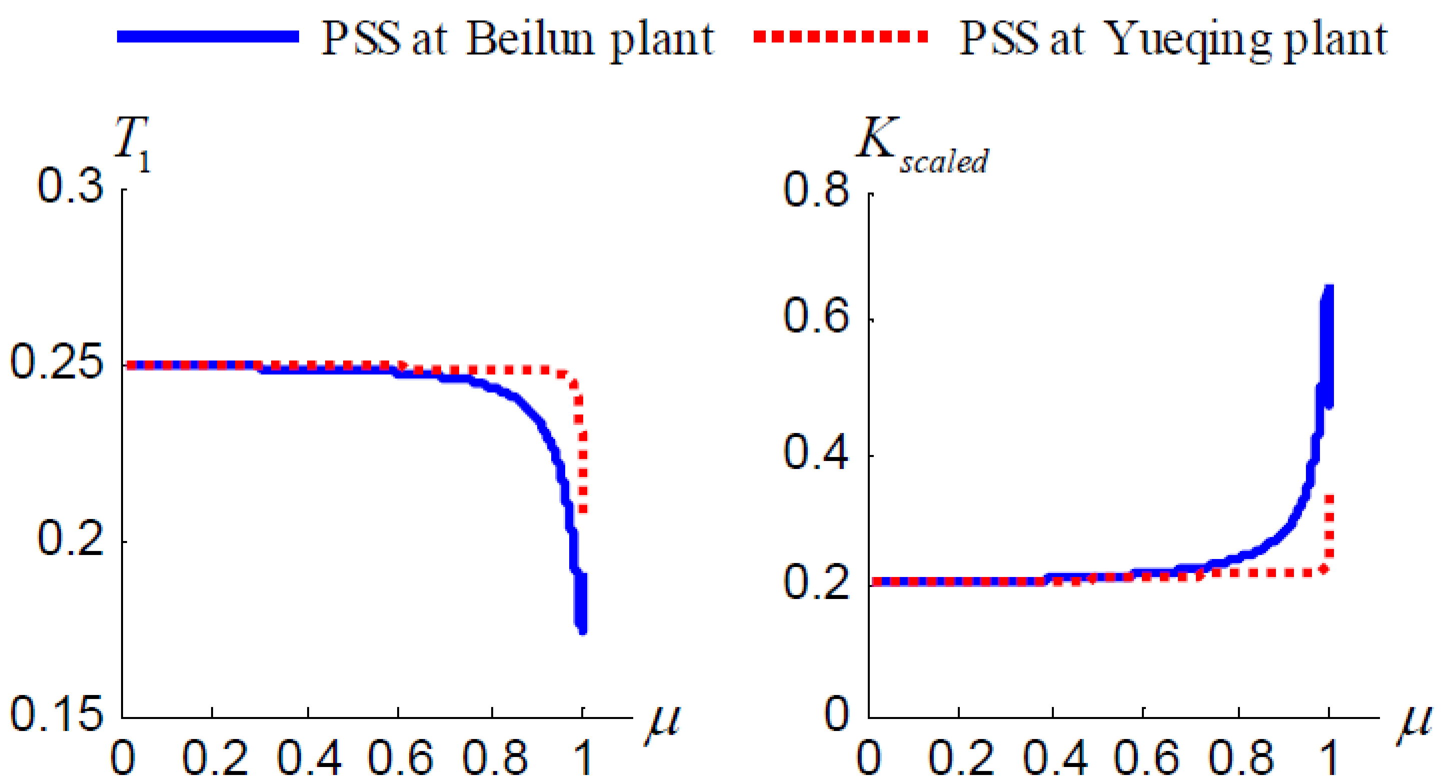
- PHM has been tested on **two real-world systems** and still works well.
- **Case 1: 1648-bus system**. 3 PSS are installed and optimized:



(a) The curve of T_1 (b) The curve of K rescaled by 100

Applicability to real-world systems

- **Case 2: East China System** (consisting of 465 machines, 3296 buses, 4559 branches including a 1000-kV ultra-voltage transmission line)



(a) The curve of T_1

(b) The curve of K rescaled by 100

Application of algebraic geometry theory

- ***Central Theorem***

Solving simultaneous multi-variate polynomial systems = Solving a series of eigenvalue problems

- ***Solving Power System Equations***

Key Point —CONSTURCT→ The multi-variate polynomial multiplication matrices

The eigenvalues of multiplication matrices = The solutions of the polynomial equations

- ***4 Steps***

① Compute the Groebner basis
② Form multiplication matrices
③ Calculate the eigenvalues
④ Match the eigenvalues

- ***Groebner Basis Theory***

Consider the following polynomial equation system:

$$f_1(x_1,\dots,x_n) = f_2(x_1,\dots,x_n) = \dots = f_s(x_1,\dots,x_n) = 0$$

The solution set of the above system is called an affine variety, which can be written as $V(f_1,\cdots,f_s)$.

The set of polynomials, denoted as $\langle f_1,\dots,f_s\rangle$, is called an ideal if

$$\langle f_1,\dots,f_s\rangle = \{p_1 f_1,\dots,p_s f_s : p_i \in k[x_1,\dots,x_n], i=1,\dots,s\}$$

Ideal $\langle f_1,\dots,f_s\rangle$ is often denoted as *I*, and $\{f_1,\dots,f_s\}$ is a basis of *I*.

- ***A simple example***

Consider a system $\begin{cases} 3x^2\text{-}2xy\text{-}y=0 \\ \text{-}x^2+2y^2\text{-}3x=0 \end{cases}$, whose Groebner basis under lexicographic order is as $\begin{cases} g_1=\text{-}5y^2\text{-}27y\text{-}48y^3+28y^4 \\ g_2=\text{-}87y^2+11y+14y^3+126x \end{cases}$.

To accomplish the mission, we solve for *y* from $g_1 = 0$.

Substituting the solution into $g_2 = 0$, we obtain the solutions for *x*. This step is called backward substitution.

- ***Quotient rings***

The following definitions are given to illustrate some basic ideas.

① Let $I \subseteq k[x_1,\ldots,x_n]$ be an ideal, $f, g \in k[x_1,\ldots,x_n]$, iff $f - g \in I$,then we say that *f* and *g* are congruent modulo *I*, written as $f \equiv g \bmod I$.

② The equivalent class of *f* modulo *I* is the set:

$$[f] = \{g \in k[x_1,\ldots,x_n] : g \equiv f \bmod I\}$$

③ The quotient of $k[x_1,\ldots,x_n]$ modulo *I*, written as $k[x_1,\ldots,x_n]/I$, is the set of equivalence classes:

$$k[x_1,\ldots,x_n]/I = \{[f] : f \in k[x_1,\ldots,x_n]\}$$

④ Let $f, g \in k[x_1,\ldots,x_n]/I$, define the sum and product operations on equivalent classes as

$$[f]+[g]=[f+g]$$

$$[f]?[g] \quad [f?g]$$

Then the quotient $k[x_1,\ldots,x_n]/I$ is a commutative ring.

- ***Normal Set of a Groebner basis***

Let $LT(I)$ denotes the leading terms of elements of *I*, the normal set of a Groebner basis is $\{x^a : x^a$ 厢 $LT(I)?\}$

- ***Calculate the values of variables***

① Let ideal *I* denote the ideal generated by the power flow equations.

Let *G* be the Groebner basis of ideal *I* with respect to any monomial order, $B = \{t_1, \cdots, t_m\}$ be the corresponding normal set.

Let $\overline{f}^G$ denote the remainder of polynomial *f* on division by the ordered *s*-tuple $G = \{g_1, ..., g_s\}$.

② Notice that

If *I* is finite-dimensional, for any $i, i = 1,\ldots,m$, we have

The above relationship can be expressed in a matrix form:

The eigenvalues of $M_{x_1},\ldots,M_{x_n}$ are exactly the roots of polynomial system generated by $G = \{g_1,\ldots,g_s\}$.

- ***A simple test system***

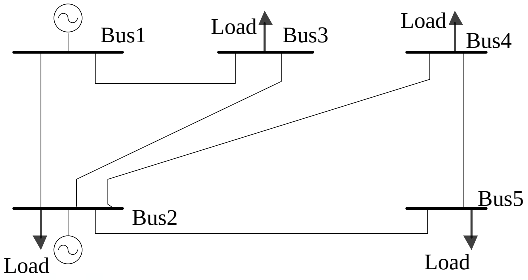


Fig. 2. The single-line diagram of a 5 bus test system

Table 1. Specified power flow data

x_1	y_1	$\lvert V_2 \rvert$	P_2	P_3
1.05	0	1.1	3	-2
Q_3	P_4	Q_4	P_5	Q_5
1	-2	1	-1.5	-0.75

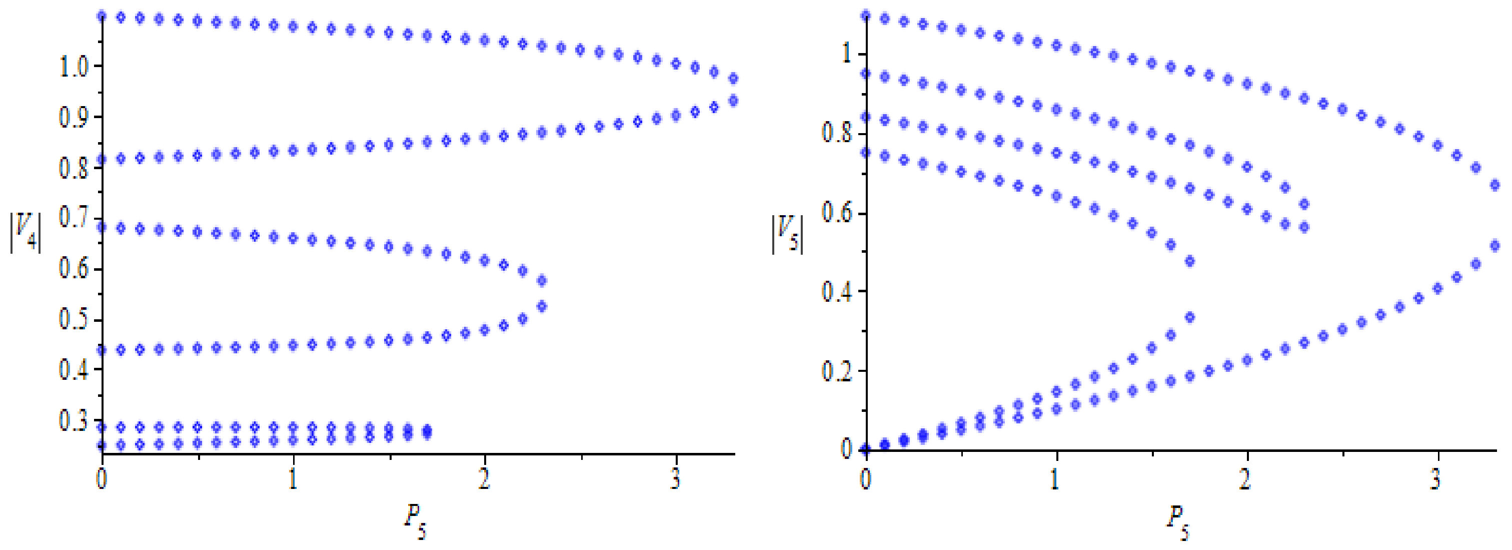
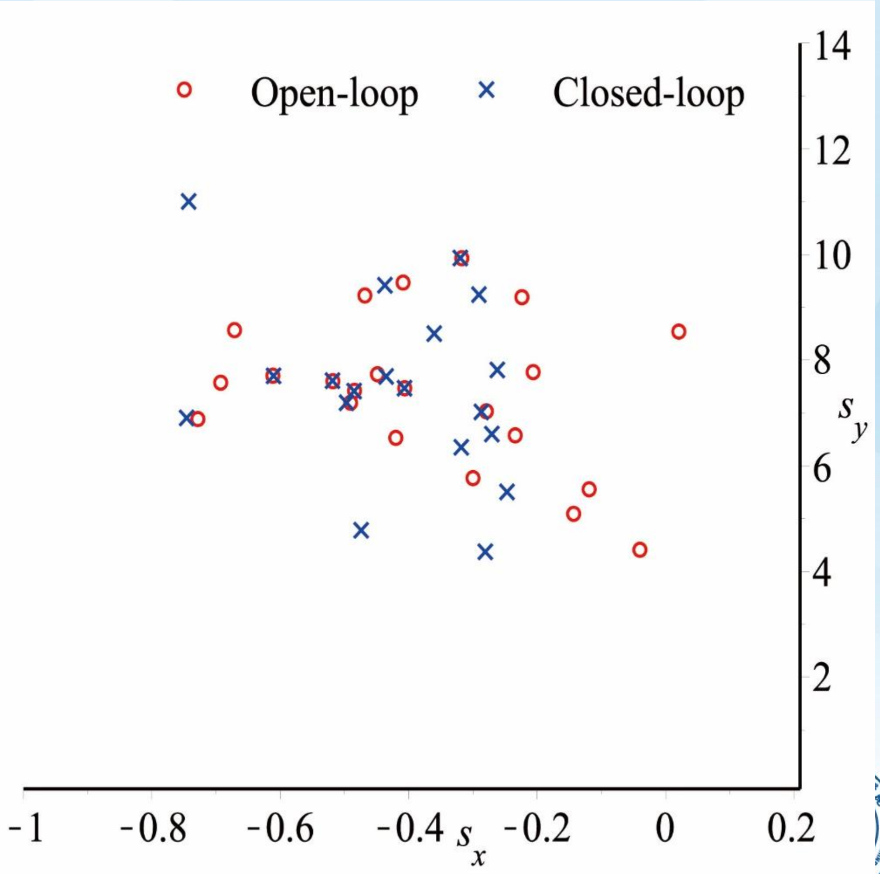


Fig. 3 PV curves

The same idea applies to PSS parameter tuning!

A eigenvalue value problem is a polynomial equation problem:

Au=su



A comparison between residue method and ideal phase curve method

Abstract

- PSS parameter tuning plays a key role in PSS's effectiveness.
- As **two most common methods** for stabilizer parameter tuning, the ***ideal phase curve method*** and ***the residue method*** are compared in:

Phase compensation

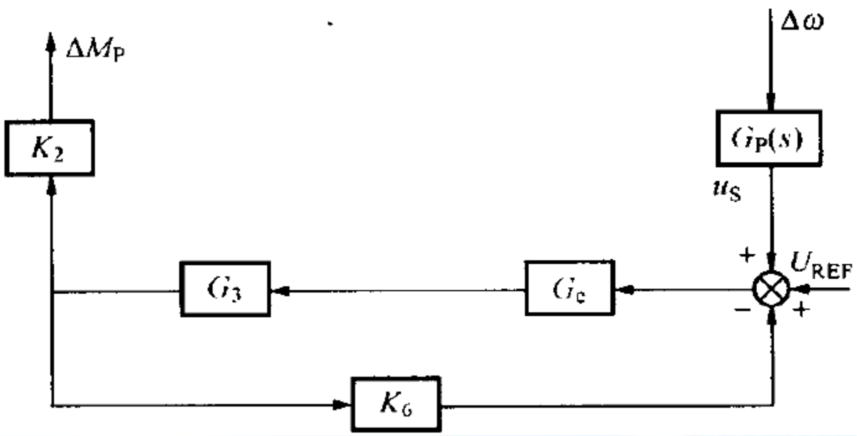
Performances in damping **local-, inter-area modes**

Ideal phase curve method

Residue method

Ideal phase curve method

- To provide damping, a PSS is supposed to **produce a component of electrical torque in phase with speed variations** of the generator.
- Thus, PSS transfer function (TF) needs to **compensate for the phase lag between the exciter input (Vr) and the electrical torque (P)**.
- This phase lag characteristic $\Delta P(j\omega)/\Delta Vr(j\omega)$ is called the ***ideal phase curve***.



Ideal phase curve method

The residue method

- **Residues** are a powerful tool for computing **eigenvalue sensitivities**.

$$\frac{\partial \lambda_j}{\partial K_{PSSi}} = R_{i,j} \cdot \left. \frac{\partial G_{PSS}(s, K_{PSSi})}{\partial K_{PSSi}} \right|_{s=\lambda_j}$$

- Hence, **the residue's ($R_{i,j}$) phase indicates the phase compensation required** so that the eigenvalue moves to the left, its **magnitude** implies the **influence of the generator** on the eigenvalue.
- Therefore, PSS's parameters can be tuned as follows:

$$\min \sum \mathrm{w}_j \cdot \mathrm{Re}(\Delta\lambda_j)$$

$$\mathrm{Re}(\Delta\lambda_j) = \mathrm{Re}\left(\frac{\partial \lambda_j}{\partial K_{PSSi}} \cdot K_{PSSi} \right)$$

Residue method

Case study

- 2 cases are studied: 162-bus, 2383-bus.

Case 1: 162-bus system	Case 2: 2383-bus system
• 25 machines • 4 unstable **inter-area modes**	• 188 machines • 2 unstable **local modes** • 1 unstable **inter-area mode**

- PSS model: PSS1A, PSS4B.
- For comparison, **the 2 methods are applied on the same set of PSSs**, with **PSS gains set at the same level**.

Case study

- Results of PSS performances on damping different modes.

	162-bus	2383-bus	
	4 Inter-area modes	2 local modes	1 inter-area mode
Ideal Phase Curve Method	X	√	√
Residue Method	√	√	√

- Both methods are **successful** for **local modes**.
- However, for **inter-area modes**, ***Ideal Phase Curve Method's* performance seems to be "erratic"**.

Case study

- Take a closer look at the **4 inter-area mode where *Ideal Phase Curve Method* is unsuccessful**:

Mode (162-bus)	Residues (Gen 5)	P-Vr phase (Gen 5)	Optimal phase compensation
54 0.02±j8.51	$0.192\angle 85°$	$-2.5°$	$95°$
56 -0.04±j4.38	$0.00004\angle -162°$	$-1.2°$	$-18°$
61 -0.14±j5.06	$0.00066\angle -140°$	$-1.5°$	$-40°$
63 -0.12±j5.53	$0.00006\angle 40°$	$-1.6°$	$-140°$

- The **P-Vr phases** for the 4 inter-area modes **are far different from the compensation phases indicated by residues**. This explains why ***Ideal Phase Curve Method*** fails here.
- On the other hand, PSSs designed by ***Residue Method*** **works quite well** on these 4 modes.

Case study

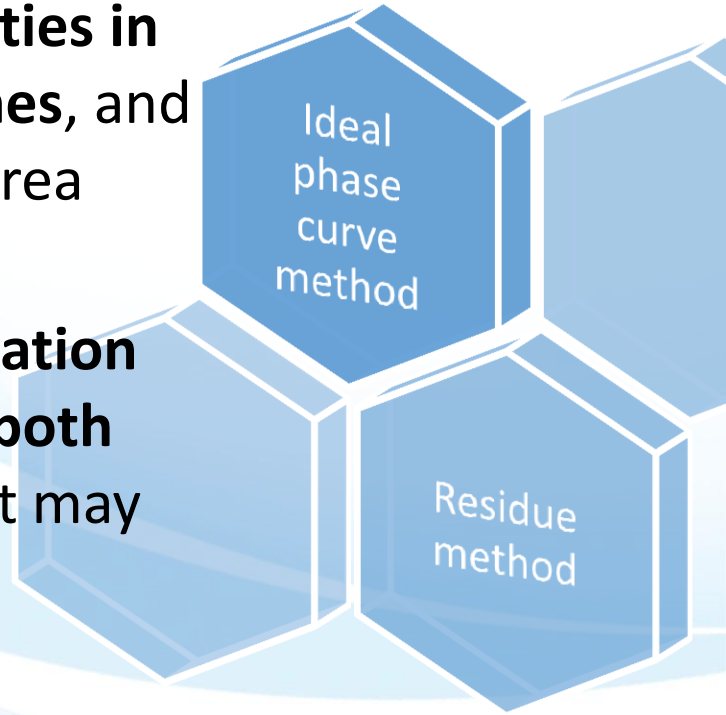
- Take a closer look at the **1 inter-area mode where *Ideal Phase Curve Method* is SUCCESSFUL**:

Generator	Residues with respect to the inter-area mode	Phase of PSS4B TF designed by *Ideal Phase Curve Method*	Phase of PSS4B TF designed by *Residue Method*
13	$0.0010\angle 149°$	2.3°	31°
32	$0.0023\angle 156°$	0.8°	24°
33	$0.0010\angle 99°$	69°	81°
37	$0.0016\angle 121°$	74°	59°
120	$0.0008\angle 133°$	0.7°	47°
123	$0.0011\angle 148°$	0.6°	32°

- ***Ideal Phase Curve Method*** provides **less ideal compensation** than ***Residue Method***, but **since the phase differences are relatively small**, it still can move this inter-area mode to the left half plane.

Other Comparison

- The ***Ideal Phase Curve (P-Vr curve)*** **keeps rather invariant over a wide range of operation conditions**, since it's determined primarily by the excitation system and the electrical circuits of the generator. (Robust)
- ***Ideal Phase Curve Method*** **faces difficulties in managing interactions between machines**, and therefore is not good at damping inter-area modes.
- ***Residue Method*** **provides useful information for PSS siting**, and is **more effective for both local- and inter-area modes**. (although it may not be that robust.)



PSS in changing operating conditions

Motivation

The network expands, which introduces oscillations.
The wind blows, which introduces changing conditions.
We can strength the network, but this costs.

Oscillating units are in different control centers!

Overview of methodologies

	Step1	Step2	Step3	
Simultaneous stabilization		Setup complete, multiple models $G_1(s),\ldots,G_k(s)$	Perform simultaneous optimization	Works in an off-line fashion, controls are computed (not designed)
Robust control		Setup a complete frequency-domain model or a polytope model $M(s)-\Delta(s), \lVert\Delta(s)\rVert \le 1$	Perform order reduction first, then design a control	Works only if the degree of uncertainty is small.
Self-tuning control	Probing is often needed	Identify a low-order model $A(z^{-1})y(k)=B(z^{-1})u(k)+e(k)$	Perform control design using pole-shifting, or optimal control, etc.	Requires accurate identification, controllers interactions not clear

Sponsors:

- ✓ Alstom, China Technology Center
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Thank you!